COURSES

derived from

The Common Curriculum Framework

for

K–12 MATHEMATICS

Grade 10 to Grade 12

Western Canadian Protocol for Collaboration in Basic Education

Call for Resources

JUNE 1996

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APPLIED MATHEMATICS 10

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The Common Curriculum Framework

for

K–12 MATHEMATICS

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APPLIED MATHEMATICS 10: GENERAL OUTCOMES, AND SPECIFIC OUTCOMES WITH ILLUSTRATIVE EXAMPLES, ORGANIZED BY STRAND AND SUBSTRAND

This section elaborates on the general outcomes and specific outcomes by providing illustrative examples, by strand and substrand, for the Applied Mathematics 10 course.

The coding for mathematical processes follows the same scheme as in the *Common Curriculum Framework*.

CLUSTERS IN THE APPLIED MATHEMATICS 10 COURSE

There are 5 clusters identified, each representing 20 to 25 hours of instructional time for an average student taking the cluster.

Common clusters, numbered C1 to C3, are part of the mathematics expected of all students completing a K to 12 mathematics program.

Applied clusters, numbered A1 to A2, emphasize applications of mathematics rather than precise mathematical theory. The approaches used are primarily numerical and geometrical.

CODING FOR ILLUSTRATIVE EXAMPLES (IEs)

The illustrative examples (IEs) listed on the following pages are organized by strand and substrand and have been correlated to specific outcomes (SOs). The numbers are taken directly from the *Common Curriculum Framework*.

NUMBERING SYSTEM

The specific outcomes are cross-referenced to the General Outcomes and Specific Outcomes section (pages 30 to 59 of the *Common Curriculum Framework*). For example, C2 – 6. (PR53) is the 6th specific outcome in Common Cluster 2 and the 53rd specific outcome in the Patterns and Relations strand.

Strand: Number (Number Concepts)

Students will:

- use numbers to describe quantities
- represent numbers in multiple ways.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

<table>
<tr><th>General Outcomes</th><th>Specific Outcomes</th><th>Illustrative Examples</th></tr>
<tr>
<td>Analyze the numerical data in a table for trends, patterns and interrelationships.

<i>(continued)</i></td>
<td>C1–1. (N1) Use words and algebraic expressions to describe the data and the interrelationships in a table with rows that are not related recursively (not calculated from previous data). [C, CN]</td>
<td>
1.1
<table>
<tr><th>Price</th><th>GST</th><th>PST</th><th>Total</th></tr>
<tr><td>$120.00</td><td>$ 8.40</td><td>$12.84</td><td>$141.24</td></tr>
<tr><td>$275.00</td><td>$19.25</td><td>$29.43</td><td>$323.68</td></tr>
</table>
a) What is the rate of GST?
b) What could be the rate of PST?
c) What could be the rule for calculating PST?
d) What is the total GST paid on the two items in the table?
e) What is the total PST paid on the two items in the table?

1.2 National Hockey League (NHL)
Western Conference: February 1, 1996
<table>
<tr><th></th><th>W</th><th>L</th><th>T</th><th>Points</th></tr>
<tr><td>Detroit</td><td>35</td><td>9</td><td>4</td><td>74</td></tr>
<tr><td>Colorado</td><td>26</td><td>14</td><td>9</td><td>61</td></tr>
<tr><td>Chicago</td><td>25</td><td>15</td><td>11</td><td>61</td></tr>
<tr><td>Toronto</td><td>22</td><td>19</td><td>9</td><td>53</td></tr>
<tr><td>St. Louis</td><td>21</td><td>20</td><td>8</td><td>50</td></tr>
<tr><td>Winnipeg</td><td>21</td><td>24</td><td>4</td><td>46</td></tr>
<tr><td>Vancouver</td><td>17</td><td>20</td><td>12</td><td>46</td></tr>
<tr><td>Los Angeles</td><td>17</td><td>22</td><td>11</td><td>45</td></tr>
<tr><td>Calgary</td><td>18</td><td>23</td><td>9</td><td>45</td></tr>
<tr><td>Edmonton</td><td>18</td><td>25</td><td>6</td><td>42</td></tr>
<tr><td>Anaheim</td><td>17</td><td>27</td><td>5</td><td>39</td></tr>
<tr><td>Dallas</td><td>14</td><td>24</td><td>10</td><td>38</td></tr>
<tr><td>San Jose</td><td>11</td><td>35</td><td>4</td><td>26</td></tr>
</table>
What happens to the NHL standings if wins are worth three points and ties are worth one point?
</td>
</tr>
</table>

Strand: Number (Number Concepts)

Students will:

- use numbers to describe quantities
- represent numbers in multiple ways.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[E]	Estimation and Mental Mathematics	**[T]**	Technology
		[V]	Visualization

<table>
<tr><th>General Outcomes</th><th>Specific Outcomes</th><th>Illustrative Examples</th></tr>
<tr>
<td><i>(continued)</i></td>
<td>C1–2. (N2) Use words and algebraic expressions to describe the data and the interrelationships in a table with rows that are related recursively (calculated from previous data). [C, CN]</td>
<td>
<p>2.1 The following table provides data on the repayment of a $100 000 farm loan. The farmer has negotiated for one annual payment to be made each year after harvest and for the right to make an extra payment, if the harvest is good. Use the table to answer the questions.</p>
<table>
<tr><th>Year</th><th>Opening Balance</th><th>Interest Rate (%)</th><th>Interest Charged</th><th>Regular Payment</th><th>Extra Payment</th><th>Closing Balance</th></tr>
<tr><td>1</td><td>$100 000.00</td><td>8</td><td>$8000.00</td><td>$14 902.95</td><td></td><td>$93 097.05</td></tr>
<tr><td>2</td><td>$ 93 097.05</td><td>8</td><td>$7447.76</td><td>$14 902.95</td><td></td><td>$85 641.87</td></tr>
<tr><td>3</td><td>$ 85 641.87</td><td>8</td><td>$6851.35</td><td>$14 902.95</td><td></td><td>$77 590.27</td></tr>
<tr><td>4</td><td>$ 77 590.27</td><td>8</td><td>$6207.22</td><td>$14 902.95</td><td></td><td>$68 894.54</td></tr>
<tr><td>5</td><td>$ 68 894.54</td><td>8</td><td>$5511.56</td><td>$14 902.95</td><td></td><td>$59 503.15</td></tr>
<tr><td>6</td><td>$ 59 503.15</td><td>8</td><td>$4760.25</td><td>$14 902.95</td><td></td><td>$49 360.46</td></tr>
<tr><td>7</td><td>$ 49 360.46</td><td>8</td><td>$3948.84</td><td>$14 902.95</td><td></td><td>$38 406.34</td></tr>
<tr><td>8</td><td>$ 38 406.34</td><td>8</td><td>$3072.51</td><td>$14 902.95</td><td></td><td>$26 575.90</td></tr>
<tr><td>9</td><td>$ 26 575.90</td><td>8</td><td>$2126.07</td><td>$14 902.95</td><td></td><td>$13 799.03</td></tr>
<tr><td>10</td><td>$ 13 799.03</td><td>8</td><td>$1103.92</td><td>$14 902.95</td><td></td><td>$ 0.00</td></tr>
</table>
<p>a) What is the period of the loan?
b) What is the amount of the annual payment?
c) How much of the annual payment at the end of Year 5 went toward the opening balance? Show how to determine the answer in two different ways.
d) Create an algebraic expression to find the answer in c).
e) If the interest rate went up to 11% in Year 10, how much would be owing at the end of Year 10?
f) What extra payment at the end of Year 4 would pay the loan off at the end of Year 8?</p>
</td>
</tr>
</table>

Strand: Number (Number Concepts)

Students will:

- use numbers to describe quantities
- represent numbers in multiple ways.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
Explain and illustrate the structure and the interrelationship of the sets of numbers within the real number system.	C1–3. (N3) Classify numbers as natural, whole, integer, rational or irrational, and show that these number sets are nested within the real number system. [C, R, V]	3.1 Explain why the number 1.112111211112 . . . is irrational. 3.2 Given a set of numbers, place them in their appropriate box in a nested Venn diagram. 3.3 Describe, orally and in writing, whether or not a number is irrational. 3.4 Demonstrate that a particular real number, such as $\sqrt{3}$, is rational or irrational.
	C1–4. (N4) Use approximate representations of irrational numbers. [R, T]	4.1 Compare the results of using different approximations for $\sqrt{2}$ in calculations. a) Calculate $\sqrt{2} \times \sqrt{2}$ as 1.4×1.4. b) Calculate $\sqrt{2} \times \sqrt{2}$ as 1.41×1.41. 4.2 Use a calculator to get the approximate value, to four decimal places, of $\sqrt{8}$ and of $2\sqrt{2}$.

Strand: Number (Number Operations)

Students will:

- demonstrate an understanding of and proficiency with calculations
- decide which arithmetic operation or operations can be used to solve a problem and then solve the problem.

[C] Communication
[CN] Connections
[E] Estimation and Mental Mathematics
[PS] Problem Solving
[R] Reasoning
[T] Technology
[V] Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
Use basic arithmetic operations on real numbers to solve problems.	C1–5. (N5) Communicate a set of instructions used to solve an arithmetic problem. [C]	5.1 Write a set of instructions that will allow another student to find: a) $1+2\div 3$ b) $9\times 4\div 3\times 5$ c) the reciprocal of a square root of a number, using a scientific calculator d) a 5% commission on a sale of $40 200.
	C1–6. (N6) Perform arithmetic operations on irrational numbers, using appropriate decimal approximations. [E, T]	6.1 Mahal indicates that $\sqrt{2}+\sqrt{8}$ has an approximate value of 3.16. Use estimates to show whether Mahal's answer is reasonable, and use a calculator to verify the accuracy of Mahal's answer. 6.2 Find a decimal approximation of $\left(\frac{3}{\sqrt{5}-\sqrt{2}}\right)$ to three decimal places. 6.3 Arrange the following in order of value from least to greatest: $7,\ 2\sqrt{13},\ 3\sqrt{6},\ 4\sqrt{5},\ 5\sqrt{2}$. Use decimal approximations. 6.4 Evaluate $\sqrt[3]{128}+4\left(\sqrt[3]{16}\right)$ to three decimal places. 6.5 Find the length of the base and the height of an equilateral triangle of area 24 cm^2.

Strand: Number (Number Operations)

Students will:

- demonstrate an understanding of and proficiency with calculations
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[C] Communication
[CN] Connections
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[V] Visualization

<table>
<tr><th>General Outcomes</th><th>Specific Outcomes</th><th>Illustrative Examples</th></tr>
<tr>
<td>Describe and apply arithmetic operations on tables to solve problems, using technology as required.

(continued)</td>
<td>C1–7. (N7) Create and modify tables from both recursive and nonrecursive situations. [PS, T, V]</td>
<td>7.1
<table>
<tr><th>Price</th><th>GST</th><th>PST</th><th>Total</th></tr>
<tr><td>$120.00</td><td>$ 8.40</td><td>$12.84</td><td>$141.24</td></tr>
<tr><td>$275.00</td><td>$19.25</td><td>$29.43</td><td>$323.68</td></tr>
</table>
a) Modify the table to allow for a PST of 6.5% of the price before taxes.
b) If the price after both taxes is $138.00 and PST is charged on the $120.00 price before taxes, what is the rate of PST?

7.2 In 1993, sales of a particular video game doubled every month. The game was released in May 1993 with sales of 32 000 for May. Prepare a table to illustrate the 1993 monthly sales figures. How many video games were sold in December 1993? Identify the assumptions you made when determining the solution.
In 1994, the demand for the video game peaked. Starting in January 1994, and every month thereafter, sales were cut to one quarter of what they were in the previous month. How many video games were sold in April 1994? If April 1994 was the last month of sales, how many video games were sold over the entire twelve months?</td>
</tr>
</table>

Strand: Number (Number Operations)

Students will:

- demonstrate an understanding of and proficiency with calculations
- decide which arithmetic operation or operations can be used to solve a problem and then solve the problem.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

<table>
<tr><th>General Outcomes</th><th>Specific Outcomes</th><th>Illustrative Examples</th></tr>
<tr>
<td>*(continued)*</td>
<td>C1–8. (N8) Use and modify a spreadsheet template to model recursive situations.
[PS, T, V]</td>
<td>
8.1 Modify the given template for a 10-year, $85 000 farm mortgage with fixed annual payments, to allow for a change in interest rate.
<table>
<tr><th>Year</th><th>Opening Balance</th><th>Interest Rate (%)</th><th>Interest Charged</th><th>Regular Payment</th><th>Closing Balance</th></tr>
<tr><td>1</td><td>$85 000.00</td><td>8</td><td>$6800.00</td><td>$12 667.51</td><td>$79 132.49</td></tr>
<tr><td>2</td><td>$79 132.49</td><td>8</td><td>$6330.60</td><td>$12 667.51</td><td>$72 795.59</td></tr>
<tr><td>3</td><td>$72 795.59</td><td>8</td><td>$5823.65</td><td>$12 667.51</td><td>$65 951.73</td></tr>
<tr><td>4</td><td>$65 951.73</td><td>8</td><td>$5276.14</td><td>$12 667.51</td><td>$58 560.36</td></tr>
<tr><td>5</td><td>$58 560.36</td><td>8</td><td>$4684.83</td><td>$12 667.51</td><td>$50 577.68</td></tr>
<tr><td>6</td><td>$50 577.68</td><td>8</td><td>$4046.21</td><td>$12 667.51</td><td>$41 956.39</td></tr>
<tr><td>7</td><td>$41 956.39</td><td>8</td><td>$3356.51</td><td>$12 667.51</td><td>$32 645.39</td></tr>
<tr><td>8</td><td>$32 645.39</td><td>8</td><td>$2611.63</td><td>$12 667.51</td><td>$22 589.52</td></tr>
<tr><td>9</td><td>$22 589.52</td><td>8</td><td>$1807.16</td><td>$12 667.51</td><td>$11 729.17</td></tr>
<tr><td>10</td><td>$11 729.17</td><td>8</td><td>$ 938.33</td><td>$12 667.51</td><td>$ 0.00</td></tr>
</table>
a) What alternatives are open to the farmer, if the interest rate increases?
b) What alternatives are open to the farmer, if the interest rate decreases?
8.2 Modify the template in illustrative example 8.1 to reflect a 25-year home mortgage with monthly payments that gives the customer the option of making an annual extra payment of $1500 at the end of any year. Interest is charged monthly.
</td>
</tr>
</table>

Strand: Number (Number Operations)

Students will:

- demonstrate an understanding of and proficiency with calculations
- decide which arithmetic operation or operations can be used to solve a problem and then solve the problem.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
Describe and apply arithmetic operations on tables to solve problems, using technology as required.	A2–1. (N9) Solve problems involving combinations of tables, using: • addition or subtraction of two tables • multiplication of a table by a real number • spreadsheet functions and templates. [PS, T, V]	1.1 The following is an income and expenses report for a business for the year ending December 31.
(continued)	*(continued)*	*(continued)*

	Year 1	Year 2	Year 3	Year 4	Year 5
Sales					
Laundry	$135 000	$148 000	$150 000	$148 000	$140 000
Dry Cleaning	45 000	47 000	48 000	45 000	45 000
Repairs and Sundry	10 000	11 000	11 000	10 000	9 000
Total Sales	$190 000	$206 000	$209 000	$203 000	$194 000
Operating Expenses					
Salaries and Wages	$ 94 000	$ 99 000	$101 000	$101 000	$ 96 000
Operating Supplies	22 000	24 000	25 000	24 000	23 000
Repairs and Misc.	4 000	5 000	6 000	8 000	5 000
Accounting and Legal	2 000	2 000	2 000	2 000	2 000
Advertising	2 000	2 000	2 000	2 000	2 000
Sundry	4 000	5 000	5 000	4 500	4 000
Total Operating Expenses	$128 000	$137 000	$141 000	$141 500	$132 000
Profit Before Overhead	$ 62 000	$ 69 000	$ 68 000	$ 61 500	$ 62 000
Overhead Expenses					
Rent	$ 12 000	$ 14 000	$ 16 000	$ 18 000	$ 18 000
Utilities	6 000	7 000	8 000	9 000	10 000
Insurance	3 000	3 000	3 000	3 000	3 000
Taxes and Licenses	3 000	3 000	4 000	4 000	5 000
Depreciation – Equip.	10 000	8 000	7 000	6 000	5 000
Total Overhead Exp.	$ 34 000	$ 35 000	$ 38 000	$ 40 000	$ 41 000
Profit Before Tax	$ 28 000	$ 34 000	$ 30 000	$ 21 500	$ 21 000
Income Tax	$ 7 000	$ 8 500	$ 7 500	$ 5 375	$ 5 250
Net Profit	$ 21 000	$ 25 500	$ 22 500	$ 16 125	$ 15 750

Strand: Number (Number Operations)

Students will:

- demonstrate an understanding of and proficiency with calculations
- decide which arithmetic operation or operations can be used to solve a problem and then solve the problem.

[C] Communication
[CN] Connections
[E] Estimation and Mental Mathematics
[PS] Problem Solving
[R] Reasoning
[T] Technology
[V] Visualization

General Outcomes	**Specific Outcomes**	**Illustrative Examples**
(continued)	*(continued)*	1.1 *(continued)* Enter the data from the previous page onto a spreadsheet template provided to students. 1.1.1 a) Calculate the dollar change in total sales, total operating expenses and total overhead expenses, between each year in the table. b) Which is the greatest dollar change? 1.1.2 a) Calculate the percentage change in total sales, total operating expenses and total overhead expenses, between each year in the table. b) Which is the greatest percentage change? 1.1.3 a) Determine the percentage change for each item for each year. b) Predict the figures for each type of income and expense for year 6, and predict the net profit for year 6. 1.1.4 Prepare a line graph showing the annual sales, operating expenses and overhead expenses for the five year period. Use the graph to determine which item has the greatest rate of increase, and which item has the greatest rate of decrease. 1.1.5 For the five year period, use a line of best fit procedure to determine equations of lines of best fit for total sales, total operating expenses and total overhead expenses. Use these equations to predict the values in year 6. From these values, predict the net profit in year 6. 1.1.6 Calculate the net profit as a percentage of sales for each of the five years. In which year did the net profit represent the highest proportion of sales? 1.1.7 Derive a formula relating total sales, total operating expenses, total overhead expenses, income tax and net profit.

Strand: Number (Number Operations)

Students will:

- demonstrate an understanding of and proficiency with calculations
- decide which arithmetic operation or operations can be used to solve a problem and then solve the problem.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
(continued)	*(continued)*	1.2 A banker needs to provide clients with information on foreign exchange. Use the foreign exchange chart provided, or a current chart from a newspaper, to answer the following questions. a) Calculate the cost in Canadian dollars of a refrigerator that costs $850 US. b) Calculate the cost in US dollars of an outboard motor selling in Canada for $1200. c) Hans receives a cheque for 100 Swiss francs from his uncle in Berne. How many Dutch guilders would he get for this cheque? How many Canadian dollars? d) Elsa is going on a holiday to Venezuela. She is told that she will have to pay $3.48 US for every 100 bolivars. How many bolivars will she get for $500 Canadian?

February 1, 1996

Foreign Exchange

Cross Rates

	Canadian dollar	US dollar	British pound	German mark	Japanese yen	Swiss franc	French franc	Dutch guilder	Italian lira
Canada dollar	–	1.3743	2.0762	0.9227	0.012850	1.1337	0.2686	0.8241	0.000865
US dollar	0.7276	–	1.5107	0.6714	0.009350	0.8249	0.1954	0.5997	0.000629
British pound	0.4816	0.6619	–	0.4444	0.006189	0.5460	0.1294	0.3969	0.000417
German mark	1.0838	1.4894	2.2501	–	0.013927	1.2287	0.2911	0.8931	0.000937
Japanese yen	77.82	106.95	161.57	71.81	–	88.23	20.90	64.13	0.067315
Swiss franc	0.8821	1.2122	1.8313	0.8139	0.011335	–	0.2369	0.7269	0.000763
French franc	3.7230	5.1165	7.7297	3.4352	0.047841	4.2208	–	3.0681	0.003220
Dutch guilder	1.2134	1.6676	2.5194	1.1196	0.015593	1.3757	0.3259	–	0.001050
Italian lira	1156.07	1588.79	2400.23	1066.71	14.855491	1310.64	310.52	952.72	–

Strand: Patterns and Relations (Relations and Functions)

Students will:

- use patterns to describe the world and to solve problems.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
Examine the nature of relations with an emphasis on functions.	C1–9. (PR47) Plot linear and nonlinear data, using appropriate scales. [C, V]	9.1 The mass of a beaker is recorded when the beaker contains varying volumes of ethanol. The results of the experiment are recorded in the table below.

Volume of Ethanol (mL)	Mass of Beaker and Liquid (g)
0	90
50	129
100	168
150	207
200	246

Measurements may be assumed correct to the nearest mL and to the nearest g.

Plot this data on a scatterplot, using appropriate scales, and answer the following questions.

a) Assuming that this pattern continues, determine the mass of the beaker and liquid when 250 mL of ethanol is present.

b) When a volume of 200 mL of ethanol is in the beaker, determine the mass of the ethanol alone.

c) The density of a liquid is defined as the mass of 1 mL of the liquid. Determine the density of the ethanol.

9.2 Nannook's Pizza uses the following price structure.

Diameter (inches)	Cost ($)
8	6.50
10	10.20
12	14.65
14	19.90
16	26.00

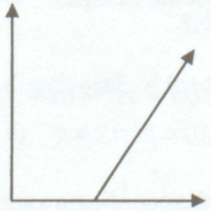
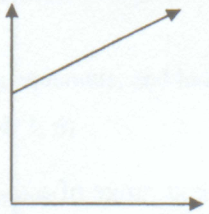
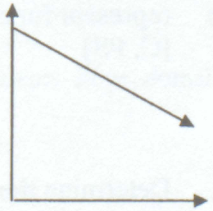
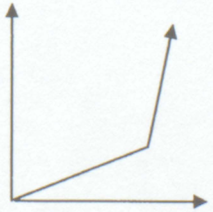
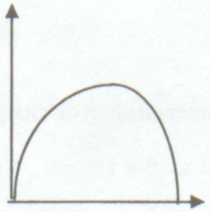
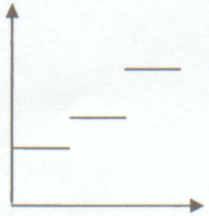
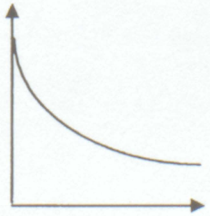
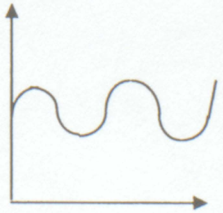
Plot this data on a scatterplot, using appropriate scales, and describe the pattern.

Strand: Patterns and Relations (Relations and Functions)

Students will:

- use algebraic and graphical models to generalize patterns, make predictions and solve problems.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[E]	Estimation and Mental Mathematics	**[T]**	Technology
		[V]	Visualization

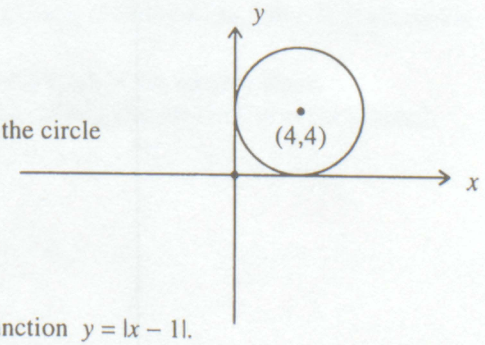
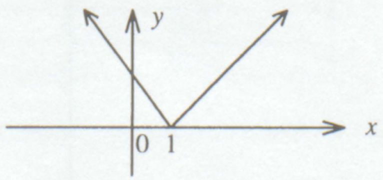
General Outcomes	Specific Outcomes	Illustrative Examples
Examine the nature of relations with an emphasis on functions. *(continued)*	C2–1. (PR48) Represent data, using function models. [CN, PS, V]	1.1 Sketch graphs to illustrate the following situations. If sufficient information is given, represent the situation by a suitable equation. Sketch and, if possible, represent by an equation: a) the area of a circle as a function of its radius b) the cost of mailing a letter as a function of the mass of the letter c) the cost of renting a car for one day as a function of the kilometres driven d) the population of Canada as a function of the year e) the length of daylight as a function of the date. 1.2 For each of the following graphs, describe a practical situation that could be represented by the graph. In describing the situation, state the meanings of any intercepts, slopes, maxima and/or minima.        

Strand: Patterns and Relations (Relations and Functions)

Students will:

- use algebraic and graphical models to generalize patterns, make predictions and solve problems.

[C] Communication
[CN] Connections
[E] Estimation and Mental Mathematics
[PS] Problem Solving
[R] Reasoning
[T] Technology
[V] Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
(continued)	C2–2. (PR49) Use a graphing tool to draw the graph of a function from its equation. [C, T, V]	2.1 Graph the function $y = x + 1$, using a graphing tool. 2.2 Graph the function $y = x^2 + 100$, using a graphing tool. Explain the process used, so that the graph appears on the screen.
	C2–3. (PR50) Describe a function in terms of: • ordered pairs • a rule, in word or equation form • a graph. [C, CN, V]	3.1 Describe the parking charges at a parkade in terms of ordered pairs, a rule and a graph.
	C2–4. (PR51) Use function notation to evaluate and represent functions. [C, PS]	4.1 If $f(x) = x^2 - 5x + 3$, find $f(2)$. What is an ordered pair describing the point on the graph having a y-coordinate of $f(2)$? 4.2 If $f(x) = 3x^2 - 6x + 5$, find $f(\sqrt{3})$, $f(2x)$ and $f(3t + 2)$.
	C2–5. (PR52) Determine the domain and range of a relation from its graph. [PS, V]	5.1 If the coordinate axes touch the circle, what is the domain and range of the circle shown in the graph to the right?  5.2 Determine, from its graph shown below, the domain and range of the function $y = \lvert x - 1 \rvert$. 

Strand: Patterns and Relations (Relations and Functions)

Students will:

- use algebraic and graphical models to generalize patterns, make predictions and solve problems.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

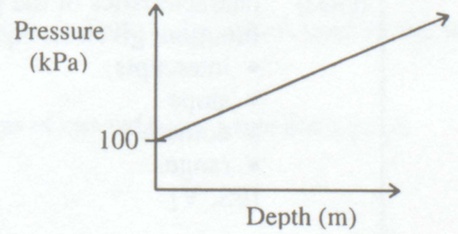
General Outcomes	Specific Outcomes	Illustrative Examples
(continued)	C2–6. (PR53) Determine the following characteristics of the graph of a linear function, given its equation: • intercepts • slope • domain • range. [PS, V]	6.1 A tanker truck drives on a weigh scale and then is filled with crude oil. The mass M, measured in kilograms, of the truck and the volume V, measured in barrels, of crude oil are related by the formula: $M = 14\,000 + 180\,V;\ V \leq 500.$ a) Draw the graph with V on the horizontal axis and M on the vertical axis. b) The tank has a maximum capacity of 500 barrels. What is the mass of the truck when it contains 500 barrels of oil? c) What is the mass of the empty truck? Where is this value found on the graph? d) Find the slope, and give an interpretation for it. e) Give the domain for this problem. f) Express the range in words. 6.2 Graph each of the following equations; and indicate intercepts, slope, domain and range. a) $y = 2x;\ x = (0, 1, 2, 3, 4, 5, 6)$ b) $y = -\frac{1}{3}x;\ x =$ a real number c) $y = 3$ d) $x = 3$ e) $y = \frac{1}{3}x + 5;\ x =$ a real number f) $y = mx + b;\ x =$ a real number

Strand: Patterns and Relations (Relations and Functions)

Students will:

- use algebraic and graphical models to generalize patterns, make predictions and solve problems.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
Represent data, using linear function models. *(continued)*	C2–7. (PR56) Use direct variation and arithmetic sequences as applications of linear functions. [CN, PS, V] *(continued)*	7.1 A hydrologist studied the relationship between the pressure on an object and its depth of submersion in a liquid. The following graph was sketched. Draw conclusions based upon the sketch.  7.2 Simple interest varies directly with the amount borrowed. a) If the interest is $5 for $100 borrowed, what would the interest be for $325 borrowed? b) Graph the relation, and write the equation of the graph. 7.3 A jet ski rental operation at Lake Okanagan charges a fixed insurance premium, plus an hourly rate. The total cost for two hours is $50 and for five hours is $110. a) Graph the relation. b) Determine the fixed insurance premium and the hourly rate to rent the jet ski. 7.4 With new equipment coming on line, a soft drink manufacturer has been increasing its production each day according to the following table. Assume a maximum daily output of 25 000 cans. Day 1 2 3 4 Units 4000 4200 4400 4600 a) Graph the relation. Hint: this is a discrete case. b) On what day will they be able to produce 20 000 cans, if this trend continues?

Strand: Patterns and Relations (Relations and Functions)

Students will:

- use algebraic and graphical models to generalize patterns, make predictions and solve problems.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
(continued)	*(continued)*	

7.5 Given the distance–time graph shown, answer the following questions.

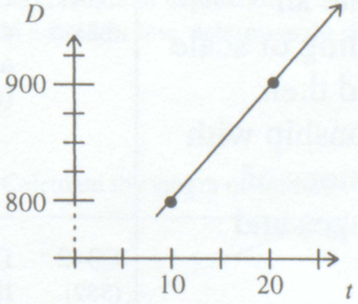
a) If $D = 850$, what is t?

b) If $t = 25$, what is D?

c) If $D = 1500$, what is t?

d) Write the equation of the function.

e) Verify the accuracy of your estimates in a), b) and c), using the equation of the function.



7.6 Given the data in the table, predict the fuel consumption for the following engines:

a) 2.5 L

b) 5.0 L.

Engine Size (L)	Consumption (L/100 km)
2.2	6.4
3.0	7.5
3.8	8.1
4.1	8.6

7.7 A video game operator gives all her change in quarters. From a \$20 bill, she gives 56 quarters change for a \$6 purchase. She gives 8 quarters change from a \$20 bill for an \$18 purchase.

a) Graph the number of quarters given as change N on the vertical axis and the amount of the purchase P on the horizontal axis. Assume that a \$20 bill was given.

b) What is the domain and range of the function?

c) How does the graph change, if a \$10 bill is used?

Strand: Shape and Space (Measurement)

Students will:

- describe and compare everyday phenomena, using either direct or indirect measurement.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[E]	Estimation and Mental Mathematics	**[T]**	Technology
		[V]	Visualization

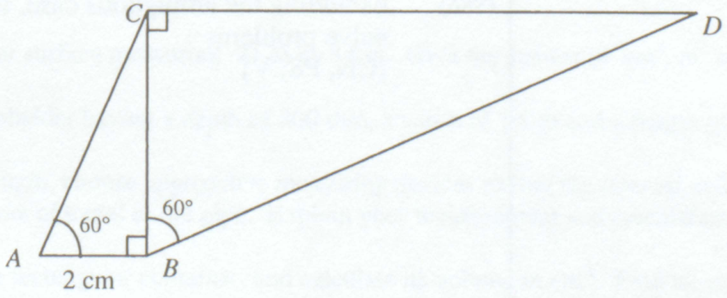
General Outcomes	Specific Outcomes	Illustrative Examples
Demonstrate an understanding of scale factors, and their interrelationship with the dimensions of similar shapes and objects.	C3–1. (SS1) Calculate the volume and surface area of a sphere, using formulas that are provided. [CN, PS, V]	1.1 Calculate the volume and surface area of a beach ball of radius 15 cm. 1.2 A hot air balloon has a spherical shape and a diameter of 4 m. If 30 additional cubic metres of air are pumped into the balloon, what will be the new values for the diameter, volume and surface area?
	C3–2. (SS2) Determine the relationships among linear scale factors, areas, the surface areas and the volumes of similar figures and objects. [CN, PS, R, V]	2.1 The area of a region in a plane is 10 cm^2. By what factor must each of the dimensions of this region be multiplied to increase the area by 20 cm^2? 2.2 A model train is built to a scale of 1:50. If the length of the model engine is 20 cm and the area of sheet metal used to cover the outside surface of the model is 180 cm^2, what is the actual length of the engine and the actual area of the sheeting used to cover the engine? If the volume displaced by the model engine is 126 cm^3, what is the volume displaced by the real engine, in m^3? 2.3 It is improbable that a giant human, 6 m in height (three or four times normal human height), could exist. Which biological systems are most likely to break down? Explain your answer.

Strand: Shape and Space (Measurement)

Students will:

- describe and compare everyday phenomena, using either direct or indirect measurement.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

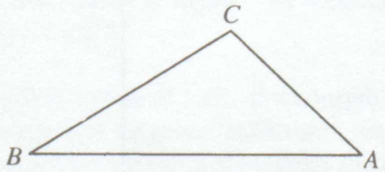
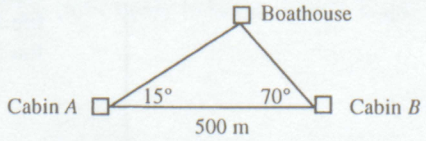
General Outcomes	Specific Outcomes	Illustrative Examples
Solve problems involving triangles, including those found in 3-D and 2-D applications.	C3–3. (SS4) Solve problems involving two right triangles. [CN, PS, V]	3.1 From the top of a 100 m fire tower, a fire ranger observes two fires, one at an angle of depression of 5° and the other at an angle of depression of 2°. Assuming that the fires and the tower are in a straight line, determine the distance between the fires for the following: a) when the fires are on the same side of the tower b) when the fires are on opposite sides of the tower. 3.2 The triangles *ABC* and *BCD* have right angles at *B* and *C* respectively. Calculate the length of side *CD*, and state the ratio of length *BD* to length *AC*.  3.3 Canada's highest waterfall is Della Falls on Vancouver Island. An observer standing at the same level as the base of the falls views the top of the falls at an angle of elevation of 58°. When the observer moves 31 m closer to the base of the falls, the angle of elevation increases to 61°. Find the height of Della Falls.
	C3–4. (SS5) Extend the concepts of sine and cosine for angles from 0° to 180°. [R, T, V]	4.1 Find sin 130°. 4.2 Use a calculator to find multiple solutions for angle *A*, if sin *A* = sin 130°. Use trial and error to find as many solutions as possible. Summarize the pattern found in the solutions.
(continued)	*(continued)*	

Strand: Shape and Space (Measurement)

Students will:

- describe and compare everyday phenomena, using either direct or indirect measurement.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[E]	Estimation and Mental Mathematics	**[T]**	Technology
		[V]	Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
(continued)	*(continued)*	4.3 Find the value(s) for A $(0° \le A \le 180°)$ when $\sin A = \frac{1}{2}$. Find the value(s) for A $(0° \le A \le 180°)$ when $\cos A = \frac{1}{2}$. Find the value(s) for A $(0° \le A \le 180°)$ when $\cos A = -\frac{1}{2}$.
	C3–5. (SS6) Apply the sine and cosine laws, excluding the ambiguous case, to solve problems. [CN, PS, V]	5.1 An electric transmission line is planned to go directly over a pond. The power line will be supported by posts at points A and B. A surveyor measures the distance from B to C as 580 m, the distance from A to C as 337 m and $\angle BCA$ as 105.34°. What is the distance from post A to post B?  5.2 Two cabins are located 500 m apart on the same side of a river. Across the river from the two cabins is a boathouse. This situation is illustrated in the diagram below. Use the measurements to find the width of the river.  5.3 A farmer has a field in the shape of a triangle. From one corner, it is 530 m to the second corner and 750 m to the third corner. The angle between the lines of sight to the second and to the third corners is 53°. Find the perimeter and area of the field. 5.4 A sailboat leaves the dock at Gibson's Landing on a bearing of S57°W. After sailing for 8 km, the ship tacks and travels S31°E for 5 km. a) How far is the sailboat from Gibson's Landing? b) What direction would it have to sail to return to the dock at Gibson's Landing? Bye et al., *Holtmath 11*, p. 313. Reprinted with permission.

Strand: Shape and Space (Measurement)

Students will:

- describe and compare everyday phenomena, using either direct or indirect measurement.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

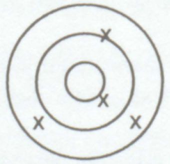
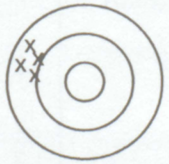
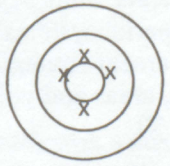
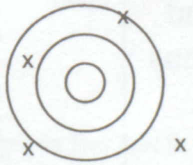
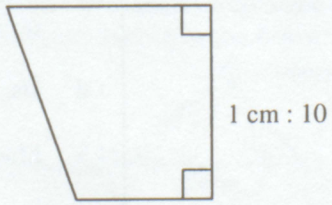
General Outcomes	Specific Outcomes	Illustrative Examples
Use measuring devices to make estimates and to perform calculations in solving problems.	A1–1. (SS8) Select and apply appropriate instruments, units of measure (in SI and Imperial systems) and measurement strategies to find lengths, areas and volumes. [E, PS, T]	1.1 Find a rule that relates hectares to acres. Is there a rule of thumb that can be used for estimates? Estimate the area of a plot of land shown in a plan, using both units of measurement. 1.2 Use a micrometer to measure the thickness of 10 sheets of paper. Use the results of this measurement to determine the thickness of one sheet of paper. 1.3 Use a micrometer to measure the thickness of a human hair. 1.4 Calculate the area of a flat rectangular surface measuring 21 m by 14 m. Give the answer in cm^2, m^2 and dm^2. 1.5 Estimate the volume of a water bed bladder having a depth of 300 mm, a width of 1.8 m and a length of 210 cm. 1.6 Given a cylindrical pipe of known length, choose appropriate measuring devices to find the internal and external diameters of the pipe. Find the volume of metal in the pipe. Explain your measurement and calculation procedures. 1.7 Measure the internal dimensions of a rectangular container, and calculate its volume in cm^3. Find its volume, in litres or in millilitres, using a calibrated cylinder. 1.8 Use a vernier caliper to measure the inside diameter of a piece of PVC pipe. 1.9 Measure the angle between two faces of a pyramid to the nearest degree. 1.10 Measure the angle of a bevel to the nearest tenth of a degree, using a vernier bevel protractor.
	A1–2. (SS9) Analyze the limitations of measuring instruments and measurement strategies, using the concepts of precision and accuracy. [C, R]	2.1 Which ruler is most precise? a) a ruler divided into tenths of an inch b) a ruler divided into eighths of an inch c) a ruler divided into millimetres.
(continued)	*(continued)*	

Strand: Shape and Space (Measurement)

Students will:

- describe and compare everyday phenomena, using either direct or indirect measurement.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

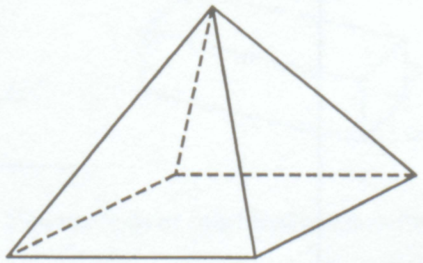
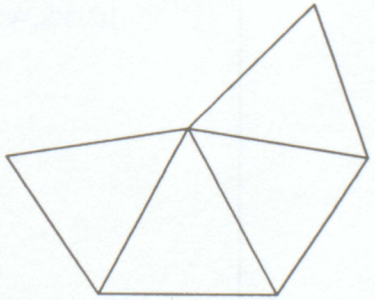
General Outcomes	Specific Outcomes	Illustrative Examples
(continued)	*(continued)*	2.2 Of the four diagrams revealing shots on a target, which best represents accuracy and precision?    
	A1–3. (SS10) Solve problems involving length, area, volume, time, mass and rates derived from these. [C, E, PS] *(continued)*	3.1 A room is 16 feet long, 12 feet wide and 8 feet high. The walls and ceiling are to be painted. There are two doors in the room, each 6 feet 6 inches high and 30 inches wide. There are two windows, each 2 feet by 4 feet. Information on the paint can states that you should allow 3.79 L for every 38 m^2 of smooth surface. Two coats of paint are needed. How many cans of paint are needed, if each can contains 3.79 L? If the painter is able to paint 3 m^2 in 10 minutes, how long will it take to paint the room? 3.2 A person buys a property that is irregularly shaped. See scale drawing below.  What is the total area, in m^2, of the lot? 3.3 A car has a highway fuel consumption of 34 miles per Imperial gallon. What is this in litres per 100 kilometres? Explain the conversion strategy used.

Strand: Shape and Space (Measurement)

Students will:

- describe and compare everyday phenomena, using either direct or indirect measurement.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[E]	Estimation and Mental Mathematics	**[T]**	Technology
		[V]	Visualization

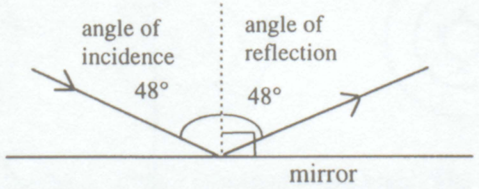
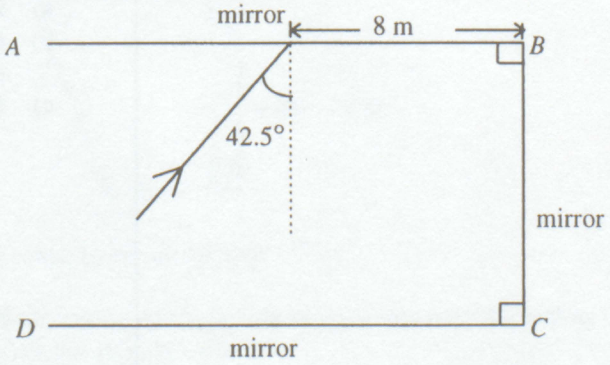
General Outcomes	Specific Outcomes	Illustrative Examples
(continued)	*(continued)*	3.4 A sheet metal worker must fabricate a pyramidal cap for a square column. The base of the cap is 1.5 m by 1.5 m and the height is 5 m. Determine the area of material required.   3.5 A building contractor is to provide wheel chair access to a new building. A space of 10 m by 10 m is available, on the west side of the entrance stairs, for a ramp. Municipal building codes specify that wheel chair ramps must have a minimum width of 1.5 m and a maximum slope of 10°. The vertical rise needed is 2 m. Construction costs for ramps of this kind average $300 per linear metre. a) Design a ramp to meet the above specifications. b) Make a plan or drawing of the proposed ramp showing the measurements, including the slopes, of the various parts. c) Give an estimate of the cost of construction.

Strand: Shape and Space (Measurement)

Students will:

- describe and compare everyday phenomena, using either direct or indirect measurement.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

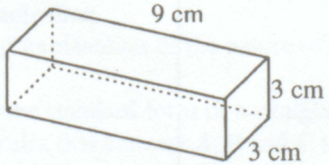
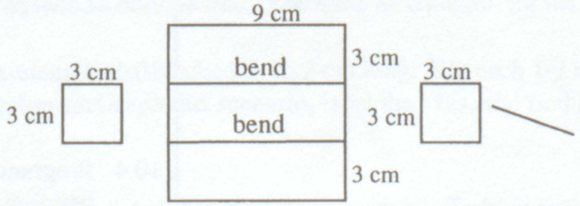
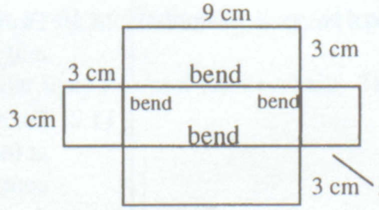
General Outcomes	Specific Outcomes	Illustrative Examples
(continued)	A1–4. (SS11) Interpret drawings, and use the information to solve problems. [C, PS, V] *(continued)*	4.1 The law of reflection states that when a ray of light is reflected at a surface, the angle of reflection is equal to the angle of incidence. Therefore, if light hits a mirror at an angle of incidence of 48°, the angle of reflection will also be 48°.  angle of incidence 48° angle of reflection 48° mirror The following diagram of the interior of a hall of mirrors shows a ray of light hitting mirror *AB* at a point 8 m from *B* and at an angle of incidence of 42.5°. Using the law of reflection, and either trigonometric relationships or scale drawings, find the angle of reflection from mirror *CD* and the distance from *C* at which the ray will hit mirror *CD*, if mirror *BC* is 12 m long.  mirror 8 m A B 42.5° mirror D C mirror

Strand: Shape and Space (Measurement)

Students will:

- describe and compare everyday phenomena, using either direct or indirect measurement.

[C] Communication
[CN] Connections
[E] Estimation and Mental Mathematics
[PS] Problem Solving
[R] Reasoning
[T] Technology
[V] Visualization

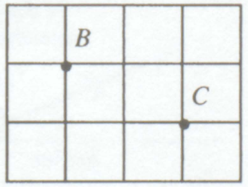
General Outcomes	Specific Outcomes	Illustrative Examples
(continued)	*(continued)*	4.2 A silver box, with dimensions as outlined below, is made from sheet metal.  Two methods of construction are shown. a)  9 cm 3 cm 3 cm bend bend 3 cm 3 cm 3 cm 3 cm ends soldered in place after bending b)  9 cm 3 cm 3 cm 3 cm bend bend bend bend 3 cm seams soldered after bending The material cost is \$2.50/cm^2, and soldering costs \$0.70/cm. For each method of construction, calculate the cost for the box.

Strand: Shape and Space (3-D Objects and 2-D Shapes)

Students will:

- describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[E]	Estimation and Mental Mathematics	**[T]**	Technology
		[V]	Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
Solve coordinate geometry problems involving lines and line segments.	C1–10. (SS19) Solve problems involving distances between points in the coordinate plane. [PS, V]	10.1 Bob and Christine want to meet; see map below. Each block has dimensions of 120 m by 120 m. Assuming the roads are of negligible width, how far does Bob B have to travel to get to Christine C? Find two separate answers, one for a path along the roads and one for a direct path.  10.2 Plot the points (–4, –2) and (1, 5) on the coordinate plane. Describe two different ways to calculate the distance between the two points. 10.3 Generate a method of determining the distance between any two points in the coordinate plane without having to plot the points. Justify your method. 10.4 Program a calculator or computer to accept, as input, the coordinates of two points and to give, as output, the distance between the two points. Document the program so that someone else can use it without assistance.
	C1–11. (SS20) Solve problems involving midpoints of line segments. [PS]	11.1 Explain to a partner the meaning of the midpoint of the line segment joining two points without using the word midpoint. 11.2 On a map with numerical coordinates in kilometres, the village of Sundown is at (6.3, 2.9), while the town of Sunup is at (4.7, 13.2). It was decided to construct a water main on the direct line joining Sunup with Sundown. Each community was responsible for the cost of construction from the community to the midpoint. Find the coordinates of the midpoint and Sundown's costs, if Sundown spent $63 475 per kilometre for construction. Determine alternative methods that could be used to solve the problem.
(continued)	C1–12. (SS21) Solve problems involving rise, run and slope of line segments. [PS, V]	12.1 If the slope of a line is 6 ($m = 6$) and the line passes through the points (2, 5) and (1, k), what is the value of k? 12.2 If two points on a line are (4, 3) and (6, 4), find one other point on the line. Use a graphing utility to demonstrate the reasonableness of your answer.

Strand: Shape and Space (3-D Objects and 2-D Shapes)

Students will:

- describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
(continued)	C1–13. Determine the equation of a line, given information that uniquely determines the line. (SS22) [PS, V]	13.1 Use a graphing device to examine changes in the graph of $y = mx + b$ as the values of m and b are changed. Use the results to explain why the equation $y = mx + b$ is called the slope and y-intercept form of a linear equation. 13.2 Write a clear explanation of the nature of the following lines: $x = a$, $y = b$, $x = y$. 13.3 Manipulate the standard form of a straight line ($Ax + By + C = 0$) into the slope and y-intercept form of the same line. Determine rules that connect A, B and C to the slope (m) and to the intercepts. 13.4 Find the equation of a line passing through the points $(-1, 3)$ and $(4, 2)$. 13.5 Given the graph of an oblique line, determine an equation for the line. 13.6 A spring with no masses attached is 25.2 cm long. For each 1-g mass attached to the spring, the spring's length increases by 4 mm. Graph this scenario, label the axes, and find an equation for the graph.
	C1–14. Solve problems using slopes of: (SS23) • parallel lines • perpendicular lines. [CN, PS, V]	14.1 Graphically examine the slopes of various lines, all of which are perpendicular to the line $y = \frac{2}{3}x + 2$. Describe the slopes, and make a rule for finding the slope of a perpendicular to a given line. 14.2 Two perpendicular lines intersect on the x-axis. The equation of one of the lines is $y = 2x - 6$. Find the equation of the second line.

Strand: Statistics and Probability (Data Analysis)

Students will:

- collect, display and analyze data to make predictions about a population.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[E]	Estimation and Mental Mathematics	**[T]**	Technology
		[V]	Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
Implement and analyze sampling procedures, and draw appropriate inferences from the data collected.	C3–6. (SP1) Choose, justify and apply sampling techniques that will result in an appropriate, unbiased sample from a given population. [C, PS, R]	6.1 A toothpaste company advertises that three out of four dentists prefer their product. Analyze this statement for its completeness and its accuracy in terms of population, sample, possible sampling technique, validity and bias. 6.2 A school cafeteria wants to introduce a new dessert. Describe how a survey could be conducted to decide which of three choices should be the new dessert. 6.3 To predict a winner in a federal election, a magazine compiled a list of about 200 000 names from sources, such as telephone books, lists of automobile owners, club membership lists and its own subscription lists. The magazine mailed a questionnaire to everybody on the list, and 4000 returned it. The 4000 responses became the sample. Discuss the potential sources of bias.
	C3–7. (SP2) Defend or oppose inferences and generalizations about populations, based on data from samples. [C, PS, R]	7.1 To determine a preference for spending $50 in either a clothing store, an electronics shop or a restaurant, customers were surveyed one Saturday morning at the mall. Fifty-nine per cent preferred spending in a clothing store, 32% in an electronics shop and 9% in a restaurant. What generalizations can be made from these results? Does the sample adequately represent the population to be surveyed? Design a more reliable sampling method to obtain this information, and include details of the questionnaires used and the method of selecting the sample. 7.2 Search through various forms of media to find examples of generalizations that have been made about populations, based on data from samples. Do you agree or disagree with the generalizations? Explain why.

Strand: Statistics and Probability (Data Analysis)

Students will:

- collect, display and analyze data to make predictions about a population.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[E]	Estimation and Mental Mathematics	**[T]**	Technology
		[V]	Visualization

General Outcomes

Apply line-fitting and correlation techniques to analyze experimental results.

Specific Outcomes

A2–2. (SP3) Determine the equation of a line of best fit, using:

- estimate of slope and one point
- median–median method
- least squares method with technology.

[CN, PS, T, V]

Illustrative Examples

2.1 Below are the heights, in metres; and masses, in kilograms, of 13 students.

Student	Height (m)	Mass(kg)
a	1.50	51
b	1.51	56
c	1.52	54
d	1.54	58
e	1.56	56
f	1.58	62
g	1.60	91
h	1.61	65
i	1.64	66
j	1.65	70
k	1.66	71
l	1.70	74
m	1.72	74

Plot the data and determine lines of best fit, using:

a) estimation

b) median–median method

c) least squares method and a computing tool.

Calculate the slope and intercept of each of the lines, and compare the results.

2.2

Oil changes per year	3	5	2	3	1	4	6	4	3	2	0	10	7
Cost of repairs	$300	300	500	400	700	400	100	250	450	650	600	0	150

a) Use graphing technology to prepare a scatterplot. Draw a line of best fit.

b) From the line of best fit, make predictions of the repair cost with eight oil changes and with 14 oil changes.

c) How reliable are these predictions?

d) Beyond what point are the predictions unreliable?

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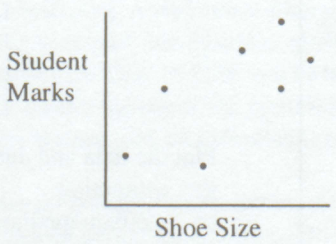
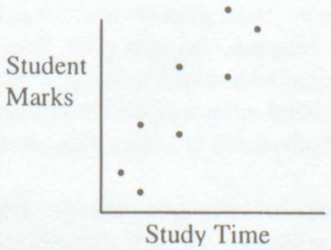
(continued)

Strand: Statistics and Probability (Data Analysis)

Students will:

- collect, display and analyze data to make predictions about a population.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[E]	Estimation and Mental Mathematics	**[T]**	Technology
		[V]	Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
(continued)	A2–3. (SP4) Use technological devices to determine the correlation coefficient *r*. [T]	3.1 Measure the height of each person in a class and the distance, from fingertip to fingertip, of their outstretched arms. a) Record this data as a set of ordered pairs, with height as the first element and fingertip to fingertip distance as the second. b) Plot the data on a coordinate system. c) By examining the data, predict a value for the correlation coefficient *r*. d) Using a calculating tool, determine the correlation coefficient *r* for this data.
	A2–4. (SP5) Interpret the correlation coefficient *r* and its limitations for varying problem situations, using relevant scatterplots. [C, R, V]	4.1 What do the following scatterplots and corresponding *r*-values represent? Scatterplot (1)  Scatterplot (2)  Scatterplot (1) is the plot of student marks on their last test against their shoe size. The value for *r* was calculated to be 0.2. Scatterplot (2) is the plot of student marks on their last test against the time spent studying. The value for *r* was calculated to be 0.8. Describe the relationship between the values of *r* and the shape of the scatterplots.

APPLIED MATHEMATICS 11

derived from

The Common Curriculum Framework

for

K–12 MATHEMATICS

Grade 10 to Grade 12

Western Canadian Protocol for Collaboration in Basic Education

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JUNE 1996

APPLIED MATHEMATICS 11: GENERAL OUTCOMES, AND SPECIFIC OUTCOMES WITH ILLUSTRATIVE EXAMPLES, ORGANIZED BY STRAND AND SUBSTRAND

This section elaborates on the general outcomes and specific outcomes by providing illustrative examples, by strand and substrand, for the Applied Mathematics 11 course.

The coding for mathematical processes follows the same scheme as in the *Common Curriculum Framework*.

CLUSTERS IN THE APPLIED MATHEMATICS 11 COURSE

There are 5 clusters identified, each representing 20 to 25 hours of instructional time for an average student taking the cluster.

Common clusters, numbered C4 to C5, are part of the mathematics expected of all students completing a K to 12 mathematics program.

Applied clusters, numbered A3 to A5, emphasize applications of mathematics rather than precise mathematical theory. The approaches used are primarily numerical and geometrical.

CODING FOR ILLUSTRATIVE EXAMPLES (IEs)

The illustrative examples (IEs) listed on the following pages are organized by strand and substrand and have been correlated to specific outcomes (SOs). The numbers are taken directly from the *Common Curriculum Framework*.

NUMBERING SYSTEM

The specific outcomes are cross-referenced to the General Outcomes and Specific Outcomes section (pages 30 to 59 of the *Common Curriculum Framework*). For example,

C2 – 6.
(PR53) is the 6th specific outcome in Common Cluster 2 and the 53rd specific outcome in the Patterns and Relations strand.

Strand: Number (Number Operations)

Students will:

- demonstrate an understanding of and proficiency with calculations
- decide which arithmetic operation or operations can be used to solve a problem and then solve the problem.

[C] Communication
[CN] Connections
[E] Estimation and Mental Mathematics
[PS] Problem Solving
[R] Reasoning
[T] Technology
[V] Visualization

General Outcomes	Specific Outcomes	Illustrative Examples	
Solve consumer problems, using arithmetic operations.	C4–1. (N12) Solve consumer problems, including: • wages earned in various situations • property taxation • exchange rates • unit prices. [CN, E, PS, R, T]	1.1	Calculate and compare wage situations involving minimum wage rates, regular pay, overtime pay, gratuities, piecework, straight commission, salary and commission, salary plus quota and graduated commission.
		1.2	Jane has a choice of two restaurants at which to work. Mario’s pays $8/h, and tips average $24 daily. Teppan’s pays $5.50/h, and tips average $35 daily. If Jane works 30 hours weekly, spread over four days, how much would she earn at each restaurant?
		1.3	Identify and calculate various payroll deductions, including income tax, CPP, UI, medical benefits, union and professional dues and life insurance premiums.
		1.4	Estimate, calculate and compare gross and net pay for various wage or salary earners in your community.
		1.5	The Ningart property has a market value of $105 000. The assessed values in the area are 60% of market values. The tax rate is 32.3 mills of assessed value. What is the Ningarts’ monthly tax payment?
		1.6	The exchange rate on a given day in the United States is 28% and in Canada 38.8%. Explain why this is possible.
		1.7	A Canadian traveller goes from Switzerland to Germany. She knows that one Swiss franc is equivalent to $1.26 Canadian (including exchange cost) and that one German mark is $0.97 Canadian (including exchange cost). How many German marks does she get for 100 Swiss francs?
		1.8	Which provides better value for tomato soup, $0.69 for 284 mL or $1.79 for 907 mL?
(continued)			

Strand: Number (Number Operations)

Students will:

- demonstrate an understanding of and proficiency with calculations
- decide which arithmetic operation or operations can be used to solve a problem and then solve the problem.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[E]	Estimation and Mental Mathematics	**[T]**	Technology
		[V]	Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
(continued)	C4–2. (N13) Reconcile financial statements including: • cheque books with bank statements • cash register tallies with daily receipts. [CN, PS, T]	2.1 The following petty cash transactions occurred during the first week of March. March 4 — $100 cheque was received to establish the fund. March 5 — Bought $12.50 worth of postage stamps. March 5 — Spent $10 to have something delivered by taxi. March 6 — Spent $6.50 for lunch. March 7 — Paid a courier service $25 for deliveries. March 7 — Bought flowers for opening day, $28. March 8 — Replenished the fund by $25. March 9 — Postage stamps purchased for $21.50. Determine if a final balance of $20 is correct. If not, provide an explanation for the difference, and indicate possible ways to correct the problem.
		2.2 Complete the table below to determine the cost of credit for using a department store charge account for the period shown. Monthly credit charges are 1.4% of the balance due.

Month	Previous Balance	− Payment Made	+ Purchases Charged	⇒ Balance Due	+ Credit Charges	⇒ New Balance
February	$314.65	$100.00	$193.75		$5.72	$414.12
March		$150.00	$ 59.60			
April		$140.00	$421.83			$618.62
May	$618.62	$200.00	$ 39.65			
June		$250.00	$ 58.11			
July		$150.00	$ 77.21			
August	$206.68	$120.00	$163.09		$3.50	$253.27

Strand: Number (Number Operations)

Students will:

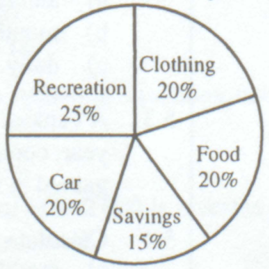
- demonstrate an understanding of and proficiency with calculations
- decide which arithmetic operation or operations can be used to solve a problem and then solve the problem.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

<table>
<tr><th>General Outcomes</th><th>Specific Outcomes</th><th>Illustrative Examples</th></tr>
<tr><td><i>(continued)</i></td><td>C4–3. (N14) Solve budget problems, using graphs and tables to communicate solutions. [C, PS, T, V]</td><td>
3.1 Research and calculate the cost of running a car for a year. Decide how to classify each cost, how to collect the data and how to display the results.

3.2 As a project, prepare a monthly budget for one of the following:
a) the family
b) an assumed persona; e.g., Wayne Gretzky
c) a school
d) a vacation
e) a fishing/hunting/shopping trip
f) a municipality.

3.3 The diagram shows Julie's monthly budget of $1200. She wants to move to her own apartment that costs $450 per month. Construct a new budget that will include her rent. Explain the choices and changes that Julie could make.

Julie Barnes' Monthly Budget


Total = $1200
</td></tr>
<tr><td></td><td>C4–4. (N15) Plot and describe data of exponential form, using appropriate scales. [C, T, V]

<i>(continued)</i></td><td>
4.1 The growth of the value of a $7000 RRSP is as follows:
<table>
<tr><th>Time (years)</th><th>Value ($)</th></tr>
<tr><td>0</td><td>7 000</td></tr>
<tr><td>1</td><td>7 630</td></tr>
<tr><td>2</td><td>8 316</td></tr>
<tr><td>3</td><td>9 065</td></tr>
<tr><td>4</td><td>9 881</td></tr>
<tr><td>5</td><td>10 770</td></tr>
</table>
Plot this data, estimate the time needed for the RRSP to reach $14 000, and determine the value of the RRSP after 12 years.
</td></tr>
</table>

Strand: Number (Number Operations)

Students will:

- demonstrate an understanding of and proficiency with calculations
- decide which arithmetic operation or operations can be used to solve a problem and then solve the problem.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[E]	Estimation and Mental Mathematics	**[T]**	Technology
		[V]	Visualization

<table>
<tr><th>General Outcomes</th><th>Specific Outcomes</th><th>Illustrative Examples</th></tr>
<tr><td><i>(continued)</i></td><td><i>(continued)</i></td><td>4.2 Plot the world population on the vertical axis and the date on the horizontal axis. Use the graph to predict the date when the population reached 4 billion and to predict the present population of the world.
<table>
<tr><th>Date</th><th>Population</th></tr>
<tr><td>1650</td><td>500 000 000</td></tr>
<tr><td>1850</td><td>1 100 000 000</td></tr>
<tr><td>1930</td><td>2 000 000 000</td></tr>
<tr><td>1950</td><td>2 500 000 000</td></tr>
<tr><td>1970</td><td>3 600 000 000</td></tr>
<tr><td>1988</td><td>5 100 000 000</td></tr>
</table></td></tr>
<tr><td></td><td>C4–5. (N16) Solve investment and credit problems involving simple and compound interest.
[CN, PS, T]</td><td>5.1 Determine the effective annual interest rate on a loan of $1000 at 10% per year, compounded quarterly.

5.2 Calculate the compound amount, after one year, of a deposit of $1000. Assume the current nominal annual interest when the interest is compounded:
a) annually
b) monthly
c) daily.

5.3 A bank offers an interest rate of 8% per year, compounded annually. A second bank offers an interest rate of 8% per year, compounded quarterly. If $2000 were deposited, for ten years, in each bank, how much more income would be gained in the second bank than in the first?

5.4 Calculate the interest paid on various forms of credit, including:
a) credit cards
b) loans
c) mortgages.

5.5 A loan of $5000 carries an interest rate of 9% per year, compounded monthly. Adele makes a payment of $350 every month. Use a spreadsheet to determine how much she still owes after making 12 payments.

5.6 Compare two investments in an RRSP for one year with contributions starting January 1.
a) $100 is invested monthly at 10% per annum, compounded monthly.
b) $600 is invested semi-annually at 10% per annum, compounded semi-annually.</td></tr>
</table>

Strand: Patterns and Relations (Variables and Equations)

Students will:

- represent algebraic expressions in multiple ways.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

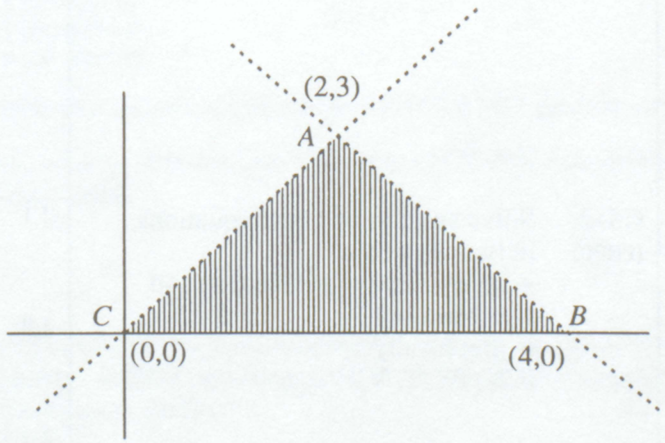
General Outcomes	Specific Outcomes		Illustrative Examples	
Represent and analyze situations that involve expressions, equations and inequalities.	C5–1. (PR29)	Graph linear inequalities, in two variables. [PS, V]	1.1 1.2	Solve, algebraically and graphically, for x: $2x + 5 > 3x - 1$. A target is described in terms of coordinates (x, y), where x and y are measured in metres. All of the following are true: • $x \leq 6$ • $y \geq 7$ • (x, y) is in the first quadrant • $x + y \leq 10$. What is the shape and the area of the target?
	C5–2. (PR30)	Solve systems of linear equations, in two variables: • algebraically (elimination and substitution) • graphically. [CN, PS, T, V]	2.1 2.2 2.3 2.4	Solve this system of equations, using the elimination method: $x + 2y = 10$ $2x + 3y = 14$. Solve this system of equations, using the substitution method: $3x + 4y = 15$ $x - y = 5$. A principal of $42 000 is invested partly at 7% and partly at 9.5%. If the interest is $3700, how much is invested at each interest rate? Plot the graphs of $2x + 3y = 11$ and $2x - 3y = 17$. What is their point of intersection?
	C5–3. (PR31)	Solve nonlinear equations, using a graphing tool. [CN, T, V]	3.1 3.2 3.3	Using a graphing tool, solve $x^2 + 6x - 11 = 0$. Solve $x^3 + x = 30$ graphically, using two different methods. Which method gives solutions that are freer from rounding errors and other inaccuracies? Where does the line $y = 4x + 5$ cut the curve $y = 2^x$? Use a graphing tool to find the points of intersection.

Strand: Patterns and Relations (Variables and Equations)

Students will:

- represent algebraic expressions in multiple ways.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[E]	Estimation and Mental Mathematics	**[T]**	Technology
		[V]	Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
Use linear programming to solve optimization problems.	A5–1. (PR36) Solve, graphically, systems of linear inequalities, in two variables, using technology. [CN, PS, T, V]	1.1 Graph the solution to the following system of inequalities: $3x - y > 4$ $2x + y \le 6$. 1.2 Given the following diagram, provide the system of inequalities whose solution is the interior of ΔABC. 
	A5–2. (PR37) Design and solve linear and nonlinear systems, in two variables, to model problem situations. [C, CN, PS, R, V]	2.1 A farmer has chickens and turkeys. He has fewer than 100 birds. He sells chickens for $10 each and turkeys for $30 each, and he earns more than $1500. Represent the situation graphically, and shade the region containing possible solutions. 2.2 A desktop publisher has to design formats for rectangular data tables and uses graphing grids as a design tool. Shade the region on the grid that represents the possible dimensions of rectangles in which the length is less than twice the width, the perimeter is at most 48 cm, and the area is at least 32 cm^2.
(continued)	*(continued)*	

Strand: Patterns and Relations (Variables and Equations)

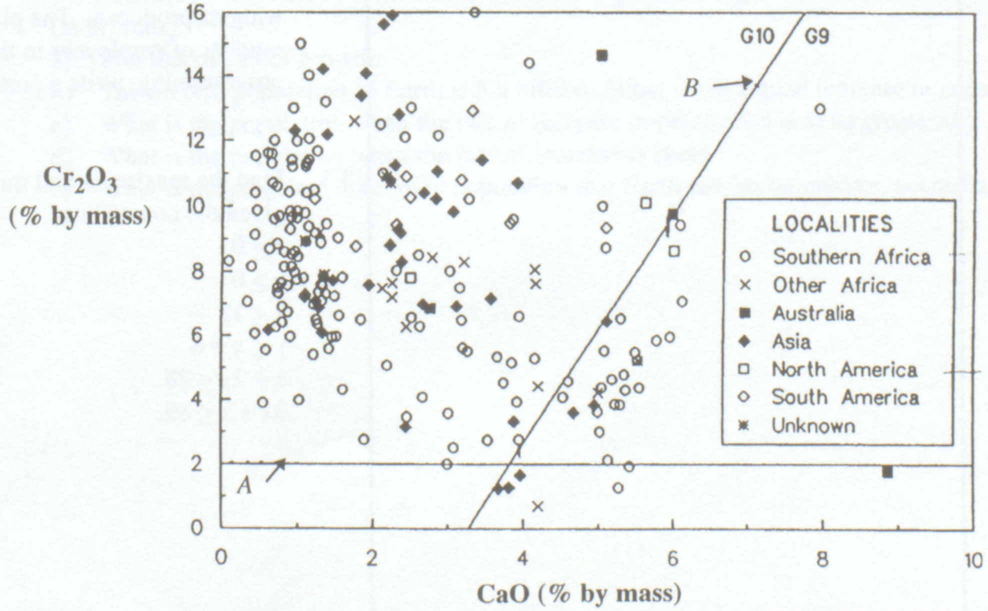
Students will:

- represent algebraic expressions in multiple ways.

[C] Communication
[CN] Connections
[E] Estimation and Mental Mathematics
[PS] Problem Solving
[R] Reasoning
[T] Technology
[V] Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
(continued)	*(continued)*	2.3 Diamond prospecting is done by testing the garnets found in rocks called kimberlites for the per cent content of Cr_2O_3 and CaO. The following graph shows the Cr_2O_3 to CaO ratio for diamond-bearing rocks worldwide. Diamonds occur 85% of the time with garnets classed as G10. This G10 area is defined by the function lines *A* and *B*. a) Define the system of linear inequalities that determines the G10 area. b) Which of the following samples would indicate that further prospecting is warranted?

Garnet Sample No.	Garnet mass (g)	Cr_2O_3 mass (g)	CaO mass (g)
1	16.1	1.71	1.35
2	8.7	0.094	0.72
3	4.2	0.35	0.051
4	12.0	1.80	0.61



Strand: Patterns and Relations (Variables and Equations)

Students will:

- represent algebraic expressions in multiple ways.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
(continued)	A5–3. (PR38) Apply linear programming to find optimal solutions to decision-making problems. [C, PS, R, T, V]	3.1 An agricultural club has a 10 ha plot of land available for a market garden project. It has selected corn and potatoes to plant and has \$4000 for the project. The corn will cost \$300/ha to grow and will generate \$375/ha gross income. The potatoes will cost \$500/ha to grow and will generate \$650/ha gross income. a) Construct the function that describes the revenue from the project. b) Construct the inequalities that describe the restrictions. c) Plot this system of inequalities. d) Identify the feasible solutions. e) Determine the optimal solution. 3.2 A manufacturing company originally has three employees. The company directive is to hire additional persons to build widgets. Widgets can only be built by teams of 2 people. Eight teams can produce 500 widgets and 10 teams can produce 600 widgets. It is assumed that a linear relation exists between the number of teams and the number of widgets produced. The plant has the capacity to produce 1000 widgets. The Department of Health limits the total number of employees in the building to 15, due to the air quality problem. Using multimedia techniques and linear programming, write a presentation to the board of directors explaining how to optimize production. 3.3 Find the maximum and minimum values of the quantity C, where $C = 2x - 5y$, given the constraints: $x \geq 0$ $y \geq 0$ $x \leq 12$ $y \leq x + 8$ $x + 2y \leq 28$ $3x + y \leq 39$.

Strand: Patterns and Relations (Relations and Functions)

Students will:

- use algebraic and graphical models to generalize patterns, make predictions and solve problems.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

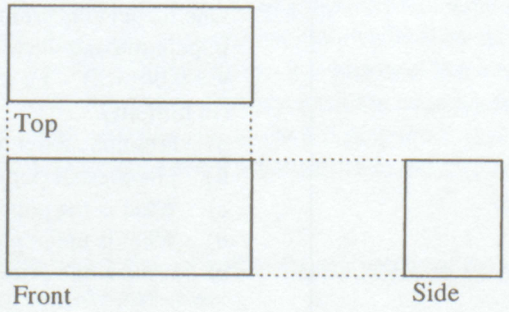
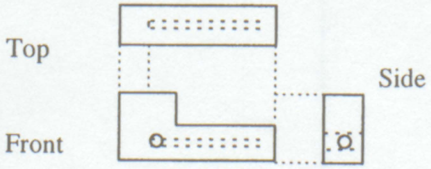
General Outcomes	Specific Outcomes	Illustrative Examples
Represent and analyze quadratic, polynomial and rational functions, using technology as appropriate.	C5–4. (PR57) Determine the following characteristics of the graph of a quadratic function: • vertex • domain and range • axis of symmetry • intercepts. [C, PS, T, V]	4.1 Given the graph of any quadratic function, determine the following: a) vertex b) domain c) range d) axis of symmetry e) intercepts. 4.2 Use technology to graph $f(x) = x^2 - 6x + 4$ and to determine the vertex, domain, range, axis of symmetry and intercepts. 4.3 One model concerning the rate of population growth of Earth has the annual rate of increase varying jointly as the population and the unused carrying capacity of Earth. The equation of the model is: $y = 0.001x(21 - x)$, where y = the rate of increase in population (in billions per year), and x = the present population (in billions). a) Plot this model of growth. b) The present population of Earth is 5.8 billion. What is the annual increase in population at present? c) What is the population when the rate of increase in population is at its greatest? d) What is the population when the rate of increase is zero? e) What is the projected maximum population that Earth can accommodate, according to this model?

Strand: Shape and Space (Measurement)

Students will:

- describe and compare everyday phenomena, using either direct or indirect measurement.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[E]	Estimation and Mental Mathematics	**[T]**	Technology
		[V]	Visualization

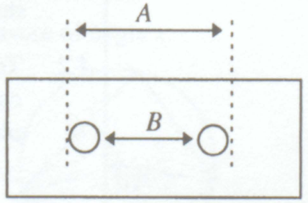
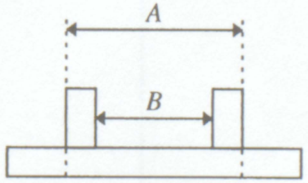
General Outcomes	Specific Outcomes	Illustrative Examples
Demonstrate an understanding of scale factors, and their interrelationship with the dimensions of similar shapes and objects.	A3–1. (SS3) Enlarge or reduce a dimensioned object, according to a specified scale. [C, CN, PS, V]	1.1 A classroom has dimensions of nine metres by eight metres. Produce a scale drawing of the classroom to a scale of 1:50. 1.2 Using surveyor's chains, tapes or other linear measuring devices, measure a chosen plot of land, and calculate its area. Make a scale drawing, using the same measurement system for the drawing as was used with the measurement instruments. 1.3 From the scale drawing below, construct an actual sized model of the box.  Scale = 1:3 1.4 To better visualize an object, architects often build clay models. Use molding clay to build a model of the object that is shown in the plan below. Scale = 2:3 

Strand: Shape and Space (Measurement)

Students will:

- describe and compare everyday phenomena, using either direct or indirect measurement.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
Use measuring devices to make estimates and to perform calculations in solving problems.	A3–2. (SS12) Calculate maximum and minimum values, using tolerances, for lengths, areas and volumes. [PS, R, V]	2.1 The diagrams represent the top and side views of a drawer handle. If the tolerance specifications are as shown below, determine the maximum and minimum dimensions for the distance between the two centres.  Figure 1: Top View  Figure 2: Side View $A = 10.50 \pm 0.02$ cm $B = 8.20 \pm 0.04$ cm
		2.2 To carry a high electric current to an LRT car, a wire must have a cross-sectional area of $45 \pm 2\text{ mm}^2$. What are the maximum and minimum diameters allowed for this wire?
		2.3 Steel ball bearings have a diameter of 0.80 ± 0.02 cm. Find the volume of one ball bearing, in cm^3, with the tolerance included. What is the maximum number of such ball bearings that can be made from 1000 cm^3 of steel?
	A3–3. (SS13) Solve problems involving percentage error when input variables are expressed with percentage errors. [PS, R, V]	3.1 A rectangular table was measured to be 420 cm long and 170 cm wide. The length was measured with an error of 1.5% and the width with an error of 2%. Calculate the maximum and minimum possible areas, and estimate the percentage error in the calculated area.
		3.2 An experiment is done to find the density of a ball bearing. The mass is measured to be 473 g, with a percentage error of 4%. The diameter is measured to be 5.1 cm ± 2%. a) Calculate the density of the ball bearing, showing its percentage error. b) Which is more effective in reducing percentage error: using a new balance that gives a mass of 473 g ± 1.5%, or using a new calliper that gives a diameter of 5.1 cm ± 1%? Justify your answer with appropriate calculations.
(continued)		

Strand: Shape and Space (Measurement)

Students will:

- describe and compare everyday phenomena, using either direct or indirect measurement.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[E]	Estimation and Mental Mathematics	**[T]**	Technology
		[V]	Visualization

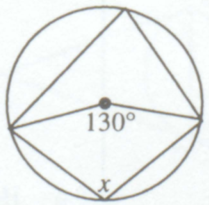
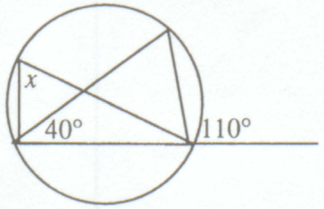
General Outcomes	Specific Outcomes	Illustrative Examples
(continued)	A3–4. (SS14) Design an appropriate measuring process or device to solve a problem. [E, PS, V]	4.1 Design and construct a measuring device; e.g., a planimeter with a horizontal vernier scale and cardboard wheel, graduated accordingly. Apply the constructed instrument to find, according to scale, the areas of large, irregular shapes. 4.2 To calculate the loss of wheat after a hailstorm, a farmer counts the number of broken wheat heads in a small area, calculates the proportion of broken heads in the sample and extrapolates this proportion to the entire field. Explain the process used to gather the data, and explain how the estimate of loss is determined.

Strand: Shape and Space (3-D Objects and 2-D Shapes)

Students will:

- describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

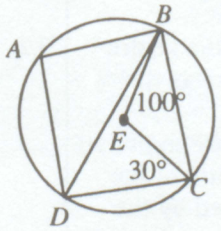
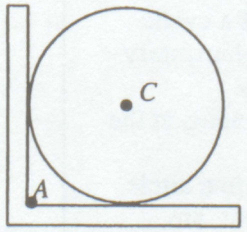
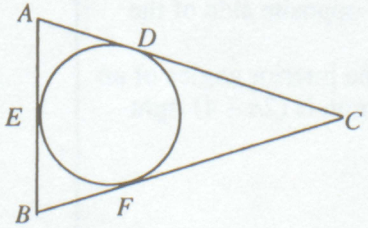
General Outcomes	Specific Outcomes	Illustrative Examples
Develop and apply the geometric properties of circles and polygons to solve problems. *(continued)*	C5–5. (SS26) Use technology and measurement to confirm and apply the following properties to particular cases: • the perpendicular from the centre of a circle to a chord bisects the chord • the measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc • the inscribed angles subtended by the same arc are congruent • the angle inscribed in a semicircle is a right angle • the opposite angles of a cyclic quadrilateral are supplementary • a tangent to a circle is perpendicular to the radius at the point of tangency • the tangent segments to a circle, from any external point, are congruent • the angle between a tangent and a chord is equal to the inscribed angle on the opposite side of the chord • the sum of the interior angles of an n-sided polygon is $(2n - 4)$ right angles. [PS, R, T, V] *(continued)*	5.1 A plate, with a diameter of 20 cm, is placed on a square place mat, with no overhang. Calculate the length of the diagonal of the square. 5.2 Determine the measure of angle x.  5.3 Determine the measure of angle x.  5.4 Draw a semicircle with diameter AB. Draw an angle, ACB, with C being any point on the semicircle. What is the measure of angle ACB? Repeat for two other points, C' and C'', on the semicircle. What pattern emerges?

Strand: Shape and Space (3-D Objects and 2-D Shapes)

Students will:

- describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

[C] Communication
[CN] Connections
[E] Estimation and Mental Mathematics
[PS] Problem Solving
[R] Reasoning
[T] Technology
[V] Visualization

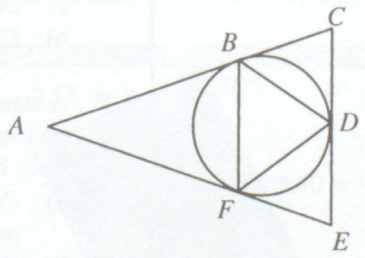
General Outcomes	Specific Outcomes	Illustrative Examples
(continued)	*(continued)*	5.5 Determine the measure of ∠ECB, ∠BDC, ∠BAD and ∠DBE, where E is the centre of the circle.  5.6 How far from the inside corner of the shelf, A, is the centre C of the plate, if the plate has a diameter of 20 cm?  5.7 The perimeter of the isosceles triangle ABC, with AC = BC, is 54 cm. If AD = 5 cm, and D, E and F are points of tangency, find the length of BC. 

Strand: Shape and Space (3-D Objects and 2-D Shapes)

Students will:

- describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
(continued)	*(continued)*	5.8 Determine the measure of $\angle CAE$, if $\angle BDF = 60°$ and $\angle FDE = 70°$. 

Strand: Shape and Space (3-D Objects and 2-D Shapes)

Students will:

- describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[E]	Estimation and Mental Mathematics	**[T]**	Technology
		[V]	Visualization

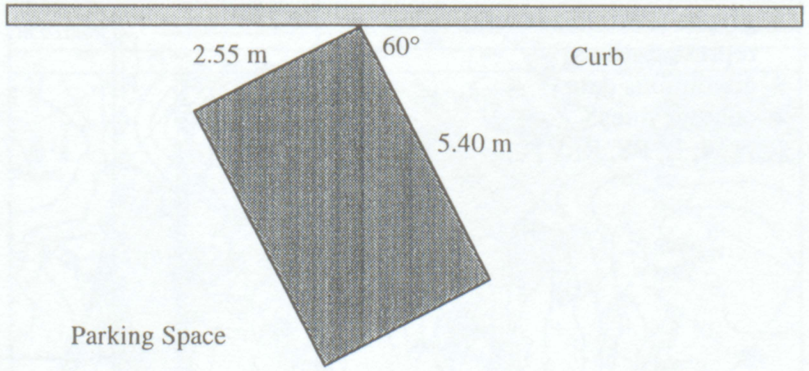
General Outcomes	Specific Outcomes	Illustrative Examples
Develop and apply the geometric properties of circles and polygons to solve problems. *(continued)*	A3–5. (SS27) Use properties of circles and polygons to solve design and layout problems. [CN, PS, V] *(continued)*	5.1 The pattern on a piece of vinyl flooring consists of a square and four equilateral triangles. Each equilateral triangle has as its base one side of the square. Circles are inscribed in each triangle and in the square. a) Start with a square of side length 6 cm. Draw the design, full size. b) Determine the ratio of the area of the small circle to the area of the large circle. 5.2 A standard sheet of paper is 22 cm by 28 cm. The margins are 3 cm on the left, on the right and at the top. The bottom margin is 4 cm. A project summary consists of one table that is 10 cm by 6 cm, three tables that are 8 cm by 5 cm each and 50 cm^2 of text that can be arranged in any shape(s). a) Prepare a possible layout, assuming that the tables can be oriented with their long sides parallel to any edge of the paper. b) Prepare a possible layout, assuming that the long side of any table must be parallel to the top edge of the paper. c) What is the maximum area of text that can be included with the four tables, if each table must have at least 1 cm margins? 5.3 A school has 325 students, all of whom have pictures to be put in the yearbook. The yearbook pages are 9.5 inches by 12 inches. The inside margins are 1.5 inches, the outside margins are 1 inch, the top margin is 1.2 inches, and the bottom margin is 1.5 inches. Each photograph is 53 mm by 35 mm. The minimum space between sides of pictures is 0.5 inches and between the bottom of one picture and the top of the next is 0.9 inches. a) How many photographs can be put on a single page? b) If the number of pages used must be divisible by 8, design a layout so that all 325 photographs can be included, without having any blank pages.

Strand: Shape and Space (3-D Objects and 2-D Shapes)

Students will:

- describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

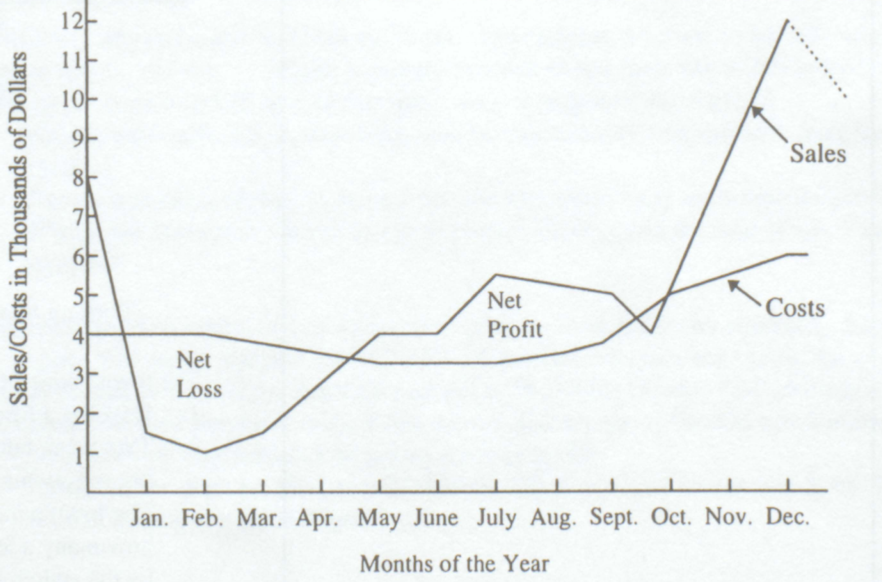
General Outcomes	Specific Outcomes	Illustrative Examples
(continued)	*(continued)*	5.4 An average automobile requires an angle parking space with dimensions of 2.55 m wide and 5.40 m long. If spaces are being calculated for parallel parking, each automobile will require an additional length of 1.20 m as manoeuvring room. A small town's main street currently uses 60° angle parking.  The town council has contracted you to provide information for town planning decisions regarding parking capacity. 1. Develop a formula for the number of spaces *N* for a given curb length *L* for 60° angle parking. 2. Two years later, increased traffic along the main street makes angle parking unsafe. The town council wants to know how many spaces *N* they will have for a given curb length *L,* if they switch to parallel parking. The town's main street is 200 m long. If the town council wants to retain the same parking capacity as before, how many additional spaces will have to be developed away from the main street in order to offset the spaces lost by the switch to parallel parking? Alberta Education, *Mathematics at Work in Alberta*, p. 9. Adapted with permission. 5.5 A cylindrical can is 12 cm high and 6 cm in diameter. The can is closed, top and bottom. It is cut from a rectangular sheet of metal, and then the pieces are sealed together to form the can. a) Determine the smallest rectangle that can be used to make one can. b) What percentage of the metal is wasted in part a)? c) If seams require 2 mm of extra metal per join, what are the new dimensions of the smallest rectangle?

Strand: Statistics and Probability (Data Analysis)

Students will:

- collect, display and analyze data to make predictions about a population.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[E]	Estimation and Mental Mathematics	**[T]**	Technology
		[V]	Visualization

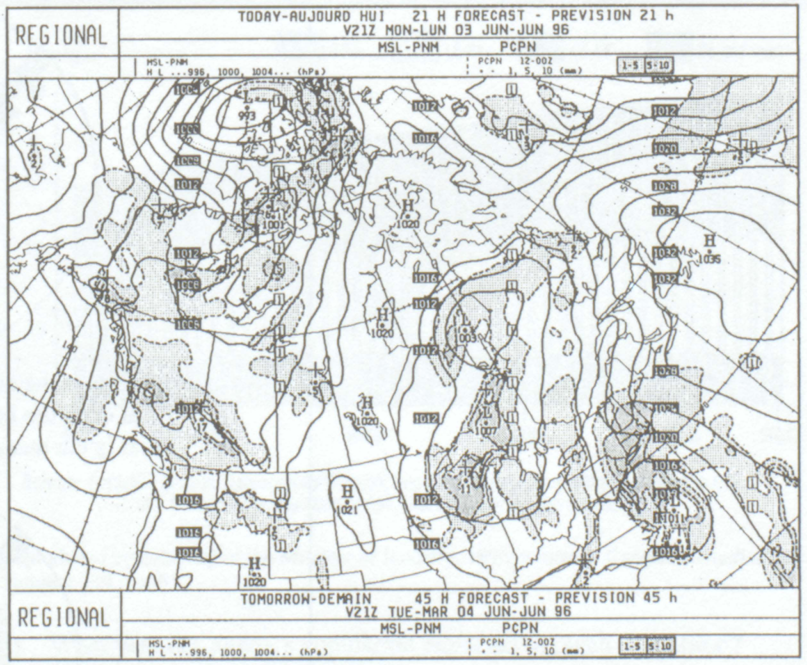
General Outcomes	Specific Outcomes	Illustrative Examples
Analyze graphs or charts of given situations to derive specific information. *(continued)*	A4–1. (SP6) Extract information from given graphs of discrete or continuous data, using: • time series • glyphs (custom pictorial representations) • continuous data • contour lines. [C, CN, E, PS, R, V] *(continued)*	1.1 Sometimes points representing discrete data are joined, even though specific values for intermediate points may not be available. Give examples where such a practice is acceptable and other examples where it is not. 1.2 PROFIT/LOSS CYCLE FOR A DEPARTMENT STORE  A department store may experience “peaks” and “troughs” in its revenue (sales). Christmas season and summer holidays are the two strongest periods. January to April can be the weakest period. If net profits are greater than net losses over the year, the business can stay in operation. a) During periods of net loss, what might the business do for finances? b) Over which of the two curves, Sales or Costs, does the business have the most managerial control? c) Discuss the net profit for May.

Strand: Statistics and Probability (Data Analysis)

Students will:

- collect, display and analyze data to make predictions about a population.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

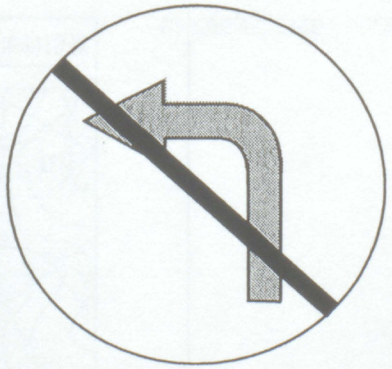
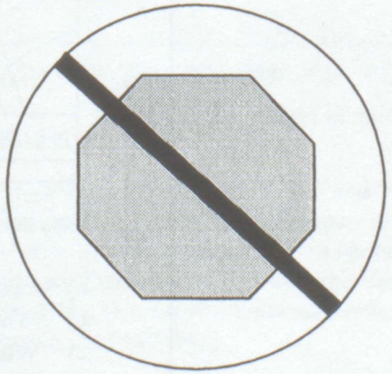
General Outcomes	Specific Outcomes	Illustrative Examples
(continued)	*(continued)*	1.3 The map below shows the atmospheric pressure, measured in hectopascals, forecast at various weather stations for June 3, 1996. A current Environment Canada map can be found on the Internet at: http://www.cmc.doe.ca/cmc/images/charts/125_100.gif  REGIONAL TODAY-AUJOURD HUI 21 H FORECAST - PREVISION 21 h V21Z MON-LUN 03 JUN-JUN 96 MSL-PNM PCPN REGIONAL TOMORROW-DEMAIN 45 H FORECAST - PREVISION 45 h V21Z TUE-MAR 04 JUN-JUN 96 MSL-PNM PCPN From Environment Canada, on line, June 2, 1996, with permission. a) Using a current map, estimate the forecasted atmospheric pressure at your location. b) What is the lowest pressure recorded in Canada for the date on your map? c) What is the highest pressure recorded in Canada for the date on your map? d) Shaded areas show where rain is falling. What connection is there between atmospheric pressure and rainfall?

Strand: Statistics and Probability (Data Analysis)

Students will:

- collect, display and analyze data to make predictions about a population.

[C] Communication
[CN] Connections
[E] Estimation and Mental Mathematics
[PS] Problem Solving
[R] Reasoning
[T] Technology
[V] Visualization

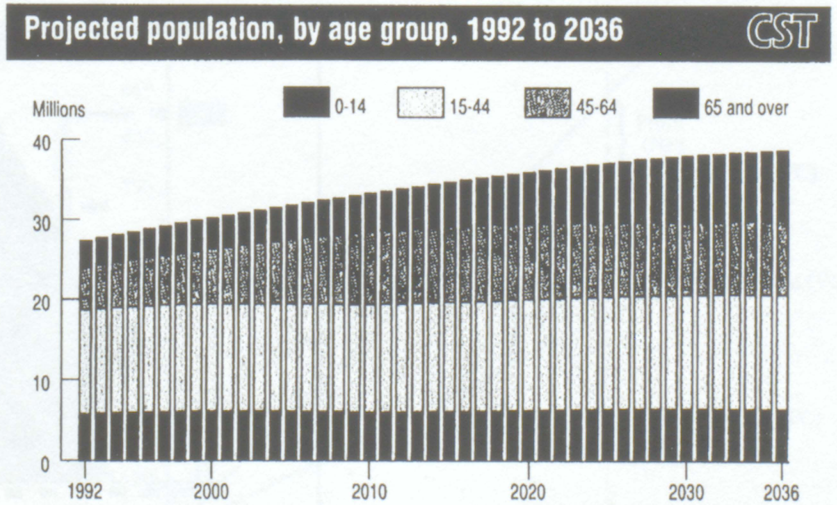
General Outcomes	Specific Outcomes	Illustrative Examples
(continued)	*(continued)*	1.4 Pictorial road signs, as used in Canada and most other countries, are examples of glyphs. They use shapes and sizes to convey the type of sign; then levels of symbols are used to convey meaning. Thus, the sign for *no left turn*, shown in the diagram below, is a two-level glyph that has a circular shape, a left turn arrow and a bar through the arrow.  a) What does the circular shape represent? b) What does the bar mean? c) What is the meaning of the sign below, and how is the meaning conveyed?  d) Design a three-level glyph for *no right turn for trucks*. Why is there no such sign in provincial operator manuals?

Strand: Statistics and Probability (Data Analysis)

Students will:

- collect, display and analyze data to make predictions about a population.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

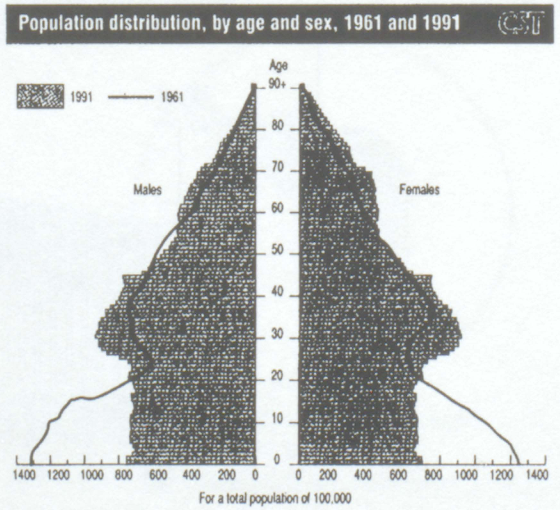
General Outcomes	Specific Outcomes	Illustrative Examples
(continued)	A4–2. (SP7) Draw and validate inferences, including interpolations and extrapolations, from graphical and tabular data. [CN, E, PS, V] *(continued)*	2.1 The bar graph below shows the projected Canadian population, by age group, for the period from 1992 to 2036.  Source: Statistics Canada, Demography Division, unpublished data, projection 3 modified to use T.F.R. of 1.84, annual immigration of 250,000, annual emigration of 86,886. Reproduced by authority of the Minister of Industry, 1996, Statistics Canada, *Canadian Social Trends*, Catalogue 11–008E, Number 29 Summer 1993, p. 6. a) What year is Canada's population expected to reach 30 million? b) Describe the rate of increase of Canada's population, both overall and by age group. c) Estimate the median age of the Canadian population in 1992 and in 2036. d) Estimate when Canada's population will reach 40 million.

Strand: Statistics and Probability (Data Analysis)

Students will:

- collect, display and analyze data to make predictions about a population.

[C] Communication
[CN] Connections
[E] Estimation and Mental Mathematics
[PS] Problem Solving
[R] Reasoning
[T] Technology
[V] Visualization

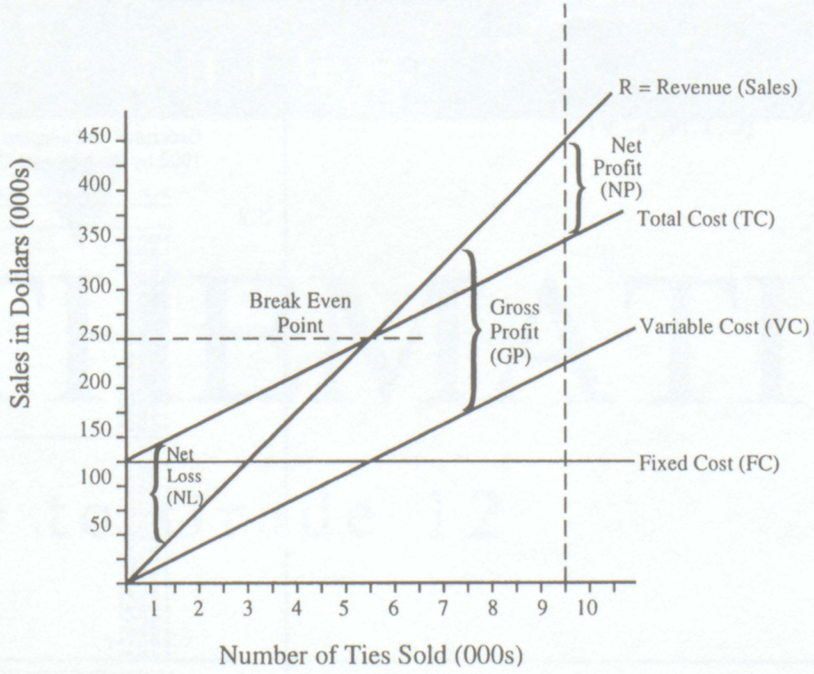
General Outcomes	Specific Outcomes	Illustrative Examples
(continued)	*(continued)*	2.2 The population pyramids shown below are for Canada for 1961 and 1991. Separate data are shown for males and females.  Population distribution, by age and sex, 1961 and 1991 Source: Statistics Canada, Demography Division. Reproduced by authority of the Minister of Industry, 1996, Statistics Canada, *Canadian Social Trends*, Catalogue 11–008E, Number 29 Summer 1993, p. 6. a) What is the approximate ratio of male births to female births? Has this ratio changed from 1961 to 1991? Describe any change, and make a hypothesis for the change. b) The baby boom was a period of time that was characterized by a greater number of births than in the years before or after. What evidence is there for a baby boom, and what were the years of the baby boom? c) The birth rate was low during the years of the Depression (1931–39) and World War II (1939–45). Where is there evidence for this? d) The shapes of the population pyramids, especially the 1961 pyramid, show a marked lack of symmetry between the data for males and the data for females. Identify where the lack of symmetry is greatest, and make hypotheses for the lack of symmetry. How could these hypotheses be tested? e) Sketch a population pyramid for the year 2011, identifying any assumptions made. Use the graph from illustrative example 2.1 as necessary.

Strand: Statistics and Probability (Data Analysis)

Students will:

- collect, display and analyze data to make predictions about a population.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General Outcomes	**Specific Outcomes**	**Illustrative Examples**
(continued)	*(continued)*	2.3 BREAK EVEN ANALYSIS  A small store in a shopping mall sells neckties for $50 each. The ties cost the merchant $25 each. Yearly operating expenses, such as wages, rent, utilities and insurance, are $125 000. $VC + FC = TC$, $R - VC = GP$, $GP - FC = NP$, $R - TC = NP$ (or NL) If the store sold 100 ties, the sales (R) would not pay for the expenses; therefore, the store would be losing money. At $250 000 in sales, the store's sales just cover all the cost of the goods sold (VC) and expenses (FC). Therefore, the store just breaks even. If the store sells 9000 ties in a year: a) What is the net profit? b) What is the gross profit? c) What is the fixed cost?

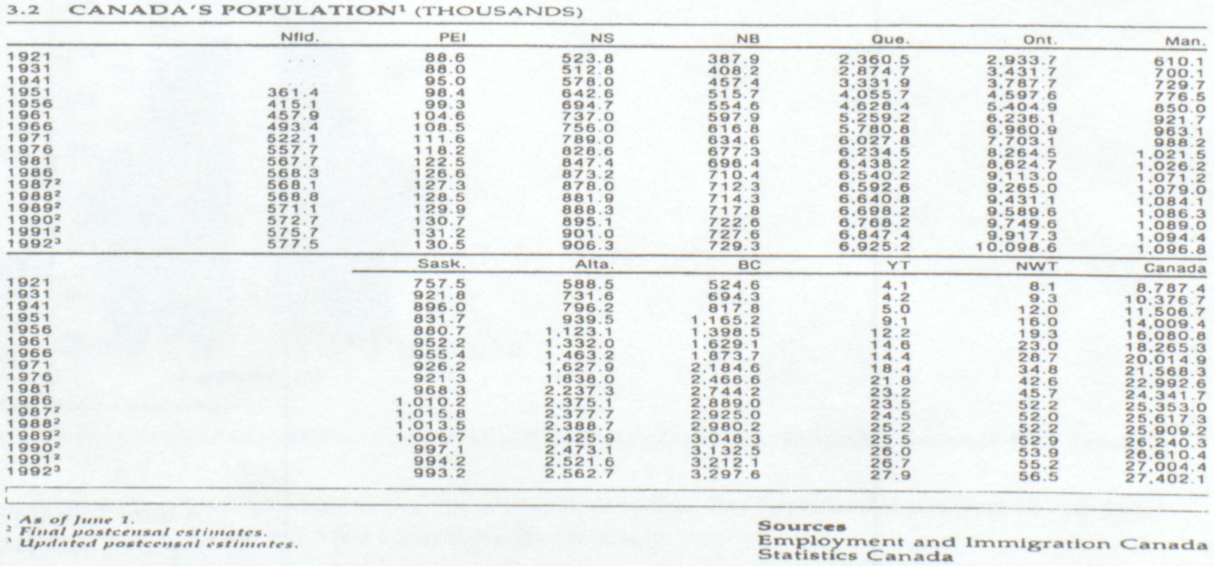
Strand: Statistics and Probability (Data Analysis)

Students will:

- collect, display and analyze data to make predictions about a population.

[C] Communication
[CN] Connections
[E] Estimation and Mental Mathematics
[PS] Problem Solving
[R] Reasoning
[T] Technology
[V] Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
(continued)	A4–3. (SP8) Design different ways of presenting data and analyzing results, by focusing on the truthful display of data and the clarity of presentation. [C, CN, T, V]	3.1 Collect an example from a newspaper or magazine in which a graph has been presented in a potentially deceptive manner. Identify the source from which the graph was taken. Explain briefly the ways in which the graph might have been deceptively presented and then show ways the data might be presented more fairly or in a less distorted fashion. Include the graph with the project, and cite its source. Excerpted and adapted with permission from *Data Analysis and Statistics* (*Curriculum and Evaluation* Addenda Series, Grades 9–12), copyright 1992 by the National Council of Teachers of Mathematics. All rights reserved. 3.2



3.2 CANADA'S POPULATION[1] (THOUSANDS)

	Nfld.	PEI	NS	NB	Que.	Ont.	Man.	Sask.	Alta.	BC	YT	NWT	Canada
1921	. . .	88.6	523.8	387.9	2,360.5	2,933.7	610.1	757.5	588.5	524.6	4.1	8.1	8,787.4
1931	. . .	88.0	512.8	408.2	2,874.7	3,431.7	700.1	921.8	731.6	694.3	4.2	9.3	10,376.7
1941	. . .	95.0	578.0	457.4	3,331.9	3,787.7	729.7	896.0	796.2	817.8	5.0	12.0	11,506.7
1951	361.4	98.4	642.6	515.7	4,055.7	4,597.6	776.5	831.7	939.5	1,165.2	9.1	16.0	14,009.4
1956	415.1	99.3	694.7	554.6	4,628.4	5,404.9	850.0	880.7	1,123.1	1,398.5	12.2	19.3	16,080.8
1961	457.9	104.6	737.0	597.9	5,259.2	6,236.1	921.7	952.2	1,332.0	1,629.1	14.6	23.0	18,265.3
1966	493.4	108.5	756.0	616.8	5,780.8	6,960.9	963.1	955.4	1,463.2	1,873.7	14.4	28.7	20,014.9
1971	522.1	111.6	789.0	634.6	6,027.8	7,703.1	988.2	926.2	1,627.9	2,184.6	18.4	34.8	21,568.3
1976	557.7	118.2	828.6	677.3	6,234.5	8,264.5	1,021.5	921.3	1,838.0	2,466.6	21.8	42.6	22,992.6
1981	567.7	122.5	847.4	696.4	6,438.2	8,624.7	1,026.2	968.3	2,237.3	2,744.2	23.2	45.7	24,341.7
1986	568.3	126.6	873.2	710.4	6,540.2	9,113.0	1,071.2	1,010.2	2,375.1	2,889.0	23.5	52.2	25,353.0
1987[2]	568.1	127.3	878.0	712.3	6,592.6	9,265.0	1,079.0	1,015.8	2,377.7	2,925.0	24.5	52.0	25,617.3
1988[2]	568.8	128.5	881.9	714.3	6,640.8	9,431.1	1,084.1	1,013.5	2,388.7	2,980.2	25.2	52.2	25,909.2
1989[2]	571.1	129.9	888.3	717.8	6,698.2	9,589.6	1,086.3	1,006.7	2,425.9	3,048.3	25.5	52.9	26,240.3
1990[2]	572.7	130.7	895.1	722.6	6,768.2	9,749.6	1,089.0	997.1	2,473.1	3,132.5	26.0	53.9	26,610.4
1991[2]	575.7	131.2	901.0	727.6	6,847.4	9,917.3	1,094.4	994.2	2,521.6	3,212.1	26.7	55.2	27,004.4
1992[3]	577.5	130.5	906.3	729.3	6,925.2	10,098.6	1,096.8	993.2	2,562.7	3,297.6	27.9	56.5	27,402.1

[1] *As of June 1.*
[2] *Final postcensal estimates.*
[3] *Updated postcensal estimates.*

Sources
Employment and Immigration Canada
Statistics Canada

Reproduced by authority of the Minister of Industry, 1996, Statistics Canada, *Canada Year Book 1994*, Catalogue No. 11–402E/1994, p. 112.

Using data for 10-year intervals, starting in 1921 and ending in 1991, design an honest presentation of the data that can be included in different term papers dealing with each of the following topics:

a) the increase in Canada's population
b) the westward shift of Canada's population
c) the population of Saskatchewan
d) the dominant position of Ontario and Quebec within Canada.

Explain your choice of data selection and data presentation.

APPLIED MATHEMATICS 12

derived from

The Common Curriculum Framework

for

K–12 MATHEMATICS

Grade 10 to Grade 12

Western Canadian Protocol for Collaboration in Basic Education

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JUNE 1996

APPLIED MATHEMATICS 12: GENERAL OUTCOMES, AND SPECIFIC OUTCOMES WITH ILLUSTRATIVE EXAMPLES, ORGANIZED BY STRAND AND SUBSTRAND

This section elaborates on the general outcomes and specific outcomes by providing illustrative examples, by strand and substrand, for the Applied Mathematics 12 course.

The coding for mathematical processes follows the same scheme as in the *Common Curriculum Framework*.

CLUSTERS IN THE APPLIED MATHEMATICS 12 COURSE

There are 5 clusters identified, each representing 20 to 25 hours of instructional time for an average student taking the cluster.

The common cluster, numbered C6, is part of the mathematics expected of all students completing a K to 12 mathematics program.

Applied clusters, numbered A6 to A9, emphasize applications of mathematics rather than precise mathematical theory. The approaches used are primarily numerical and geometrical.

CODING FOR ILLUSTRATIVE EXAMPLES (IEs)

The illustrative examples (IEs) listed on the following pages are organized by strand and substrand and have been correlated to specific outcomes (SOs). The numbers are taken directly from the *Common Curriculum Framework*.

NUMBERING SYSTEM

The specific outcomes are cross-referenced to the General Outcomes and Specific Outcomes section (pages 30 to 59 of the *Common Curriculum Framework*). For example,

$\underset{\text{(PR53)}}{\text{C2 – 6.}}$ is the 6th specific outcome in Common Cluster 2 and the 53rd specific outcome in the Patterns and Relations strand.

Strand: Number (Number Operations)

Students will:

- demonstrate an understanding of and proficiency with calculations
- decide which arithmetic operation or operations can be used to solve a problem and then solve the problem.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
Describe and apply operations on matrices to solve problems, using technology as required. *(continued)*	A6–1. (N17) Show an understanding of matrices and perform the operations of addition, scalar multiplication and matrix multiplication. [C, T]	1.1 Calculate each of the following: a) $\begin{pmatrix} 4 & 6 \\ 2 & -1 \end{pmatrix} + \begin{pmatrix} 3 & 8 \\ 2 & -5 \end{pmatrix}$ b) $4\begin{pmatrix} 2 & 3 & -4 \\ 1 & 0 & 5 \end{pmatrix}$ c) $\begin{pmatrix} 3 & 2 \\ -1 & 4 \end{pmatrix}\begin{pmatrix} 4 & 1 & -2 \\ 3 & 5 & 0 \end{pmatrix}$. 1.2 Represent a real-world situation, using a matrix. a) For towns participating in a local hockey league, create hockey standings, including home, away and combined records. b) Diagram various networking strategies, such as those found in an office, in a telephone system, in a roadway system. 1.3 Singh's Grocery sells several different kinds of breakfast cereal, each at a different price. Cereal A is 2.65/bx. Cereal B is 3.73/bx. Cereal C is 3.15/bx. Cereal D is 2.99/bx. Write the price list as a row matrix. On Wednesday, they sold the following: 5 boxes of Cereal A 8 boxes of Cereal B 7 boxes of Cereal C 10 boxes of Cereal D. Write Wednesday's sales as a column matrix. Use matrix multiplication to find Wednesday's total revenues.

Strand: Number (Number Operations)

Students will:

- demonstrate an understanding of and proficiency with calculations
- decide which arithmetic operation or operations can be used to solve a problem and then solve the problem.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[E]	Estimation and Mental Mathematics	**[T]**	Technology
		[V]	Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
(continued)	A6–2. (N18) Solve problems, using the operations of addition, subtraction, scalar multiplication and matrix multiplication on matrices. [PS, R, T, V] *(continued)*	2.1 A store sells items that are tax-free, items that have a 7% GST charge on the base price and items that have both a 7% GST and a 9% PST charge on the base price. A weekend's sales, before tax, can be represented by: $S = \begin{array}{l} \\ \text{Tax free} \\ \text{GST only} \\ \text{GST and PST} \end{array} \begin{array}{cc} \text{Saturday} & \text{Sunday} \\ \multicolumn{2}{c}{\begin{pmatrix} 500 & 700 \\ 1250 & 400 \\ 800 & 700 \end{pmatrix}} \end{array}$ a) What does the matrix $A = \begin{pmatrix} 0 & 0 \\ 1250 & 400 \\ 800 & 700 \end{pmatrix}$ represent? b) What does the matrix $B = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 800 & 700 \end{pmatrix}$ represent? c) What does the matrix $(S + 0.07A + 0.09B)$ represent? d) Write a matrix to represent the total tax collected. What are the entries for this matrix? e) Budgets change the tax rates to 5% for GST and 12% for PST. Write a new matrix for the total taxes collected in this new situation. What are the entries for this new matrix? 2.2 Sales of economy cars were 200 in 1993 and rose by 3% in 1994. Sales of midsize cars were 300 in 1993 and rose by 10% in 1994. Sales of luxury cars were 40 in 1993 and fell by 5% in 1994. Show that 1994 sales can be represented by the matrix multiplication shown. $\begin{pmatrix} 1.03 & 0 & 0 \\ 0 & 1.10 & 0 \\ 0 & 0 & 0.95 \end{pmatrix} \begin{pmatrix} 200 \\ 300 \\ 40 \end{pmatrix} = \begin{pmatrix} 206 \\ 330 \\ 38 \end{pmatrix}$

Strand: Number (Number Operations)

Students will:

- demonstrate an understanding of and proficiency with calculations
- decide which arithmetic operation or operations can be used to solve a problem and then solve the problem.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General Outcomes

(continued)

Specific Outcomes

(continued)

Illustrative Examples

2.3 Soccer has been experimenting with using league standings to discourage tie games, especially those with no goals. The traditional scheme of 2 points for a win and 1 point for any tie has been replaced by 3 points for a win and 1 point for any tie. Proposed schemes have included 3 points for a win, 1 point for ties that have goals scored and 0 points for ties with no goals; as well as a scheme with 5 points for a win, 3 points for a tie with goals scored and 0 points for a tie with no goals. In a local soccer league the top four team records after 42 games are:

	Wins	Ties with Goals	Ties with no Goals	Losses
Tigers	30	2	8	2
Irish	24	9	2	7
Colts	25	7	0	10
Jets	26	1	10	5

a) Multiply the matrix above by $\begin{pmatrix} 2 \\ 1 \\ 1 \\ 0 \end{pmatrix}$ to get the traditional points.

b) Multiply the matrix above by $\begin{pmatrix} 3 \\ 1 \\ 1 \\ 0 \end{pmatrix}$, by $\begin{pmatrix} 3 \\ 1 \\ 0 \\ 0 \end{pmatrix}$ and by $\begin{pmatrix} 5 \\ 3 \\ 0 \\ 0 \end{pmatrix}$ to get the points under the alternative systems.

c) Which of the alternative scoring systems can make the Irish second in the standings?

d) Which of the alternative scoring systems can make the Colts second in the standings?

e) Which of the alternative scoring systems can make the Jets second in the standings?

f) Design a system that would drop the Tigers out of first place. Is it a fair system?

Strand: Number (Number Operations)

Students will:

- demonstrate an understanding of and proficiency with calculations
- decide which arithmetic operation or operations can be used to solve a problem and then solve the problem.

[C] Communication
[CN] Connections
[E] Estimation and Mental Mathematics
[PS] Problem Solving
[R] Reasoning
[T] Technology
[V] Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
(continued)	*(continued)*	2.4 Diplomacy in the Asia-Pacific region is complicated by different alliances. The exchange of diplomats in 1996 can be represented by the matrix ***D***, where: $D = \begin{array}{l\|ccccc} & \text{NK} & \text{SK} & \text{Ch} & \text{T} & \text{Can} \\ \hline \text{North Korea} & 0 & 0 & 1 & 0 & 0 \\ \text{South Korea} & 0 & 0 & 0 & 1 & 1 \\ \text{China} & 1 & 0 & 0 & 0 & 1 \\ \text{Taiwan} & 0 & 1 & 0 & 0 & 0 \\ \text{Canada} & 0 & 1 & 1 & 0 & 0 \end{array}$ An entry of 1 represents an exchange of ambassadors; an entry of 0 represents no exchange of ambassadors. a) Draw a network diagram to represent the matrix. Powers of the matrix ***D*** represent the number of diplomatic channels available for the exchange of data. The matrix D^2 represents channels with one intermediary, matrix D^3 represents channels with two intermediaries, and matrix D^4 represents channels with three intermediaries. The channels can be listed after the number of channels are identified. *(continued)*

Strand: Number (Number Operations)

Students will:

- demonstrate an understanding of and proficiency with calculations
- decide which arithmetic operation or operations can be used to solve a problem and then solve the problem.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
(continued)	*(continued)*	2.4 *(continued)* b) Verify that the matrix D^2 is given by: $\begin{pmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 2 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 2 \end{pmatrix}$ Explain why there are no zero entries along the diagonal between top left and bottom right. c) Verify that D^3 is the matrix: $\begin{pmatrix} 0 & 1 & 2 & 0 & 0 \\ 1 & 0 & 0 & 2 & 3 \\ 2 & 0 & 0 & 1 & 3 \\ 0 & 2 & 1 & 0 & 0 \\ 0 & 3 & 3 & 0 & 0 \end{pmatrix}$ Trace the channel between China and Taiwan. d) The matrix D^4 is given by: $\begin{pmatrix} 2 & 0 & 0 & 1 & 3 \\ 0 & 5 & 4 & 0 & 0 \\ 0 & 4 & 5 & 0 & 0 \\ 1 & 0 & 0 & 2 & 3 \\ 3 & 0 & 0 & 3 & 6 \end{pmatrix}$ Trace out the path that a message would take to go from North Korea to Taiwan, using three intermediaries. *(continued)*

Strand: Number (Number Operations)

Students will:

- demonstrate an understanding of and proficiency with calculations
- decide which arithmetic operation or operations can be used to solve a problem and then solve the problem.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[E]	Estimation and Mental Mathematics	**[T]**	Technology
		[V]	Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
(continued)	*(continued)*	2.4 *(continued)* e) The matrix $D + D^2 + D^3$ is given by: $\begin{pmatrix} 1 & 1 & 3 & 0 & 1 \\ 1 & 2 & 1 & 3 & 4 \\ 3 & 1 & 2 & 1 & 4 \\ 0 & 3 & 1 & 1 & 1 \\ 1 & 4 & 4 & 1 & 2 \end{pmatrix}$ This matrix represents all those channels that need two or fewer intermediaries. Trace out the one channel between Canada and Taiwan and all four channels between Canada and South Korea.
	A6–3. (N19) Use matrices and matrix operations to model and to solve consumer, network and schedule problems. [C, CN, PS, R, T, V]	3.1 A washing powder is sold in 6 L and 10 L packages. Market research shows that 7% of the users of the 6 L size switch to the 10 L size for their next purchase, and 3% of the users of the 10 L size switch to the 6 L size for their next purchase. a) If the original market share was 60% for 6 L and 40% for 10 L, what is the market share for each size in the next round of purchases? b) What is the market share for each size for the third round of purchases? c) Rewrite the processes for a) and b) in terms of a 2 × 2 transition matrix and a 2 × 1 market share matrix. d) What would be the final market share? e) Use iteration to estimate how quickly the final market share for each size is approached. 3.2 A car manufacturer makes three models of car: full size, compact and economy. Of full size car buyers, 13% will switch to compact and 2% to economy. Of compact car buyers, 5% will switch to full size and 4% to economy. Of economy car buyers, 21% will switch to compact and 3% to full size. a) If the initial market share is 30% full size, 20% compact and 50% economy, what is the market share for each model for the next round of purchases? b) What is the market share for each model for the third round of purchases? c) Write a 3 × 3 matrix T that represents the switching behaviour. d) Find the final market share for each model.

Strand: Number (Number Operations)

Students will:

- demonstrate an understanding of and proficiency with calculations
- decide which arithmetic operation or operations can be used to solve a problem and then solve the problem.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
Design or use a spreadsheet to make and justify financial decisions. *(continued)*	A8–1. (N20) Design or modify a financial spreadsheet template to allow users to input their own variables. [C, PS, T]	1.1 For the following invoice, develop a spreadsheet that calculates the totals and that requires the operator to input a minimum number of entries. ACME AUTO PARTS Customer Inquiries

Item No.	Auto Parts	Quantity	Unit Price	Total	Labour	
1	Brake Pads	1	26.34	26.34	O/H Front Brakes 1.5 hrs. @ 37.00/hr.	51.25
2	Wheel Seals	2	5.25	10.50	Machined and Replaced Rotor	10.00
3	Rotor	1	30.16	30.16		
					Total Labour	61.25
					Total Parts	67.00
			Total Parts	67.00	PST on Parts (8%)	5.36
					GST (7%)	8.98
					TOTAL	142.59

Strand: Number (Number Operations)

Students will:

- demonstrate an understanding of and proficiency with calculations
- decide which arithmetic operation or operations can be used to solve a problem and then solve the problem.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[E]	Estimation and Mental Mathematics	**[T]**	Technology
		[V]	Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
(continued)	A8–2. (N21) Use spreadsheets to analyze renting or buying an increasing asset (home) under different sets of circumstances. [C, PS, T]	2.1 The Wong family is faced with a move and has the choice of buying a home for $145 000 with a $25 000 down payment, or renting a similar house for $975 per month. Four options are available. 1. Buy the house with a 20-year mortgage and continue investing at the same rate after the mortgage is paid. 2. Buy the house with a 30-year mortgage. 3. Rent a house and invest the $25 000. 4. Rent a house and invest both the $25 000 and the difference each month between the rent and the mortgage payment. The analysis spreadsheets must include the following inputs: a) mortgage interest rate, taking 8.5% as a starting value b) taxation rate, taking 1.5% of market value as a starting value c) annual rent increase, taking 5% per annum as a starting value d) annual increase in house value, taking 4% per annum as a starting value e) investment return, taking 7.0% as a starting value. Try different scenarios, varying from 1 year to 30 years. Summarize circumstances in which buying makes sense, and summarize circumstances when renting makes sense.
	A8–3. (N22) Use spreadsheets to analyze leasing or buying a decreasing asset (vehicle, computer) under different sets of circumstances. [C, PS, T]	3.1 A car lease runs for 36 months at $305 per month, with a down payment of $1105, a lease-end value of $7105 and an interest rate of 11.6%. Maintenance is the purchaser's responsibility. Set up a spreadsheet to include the monthly values of the opening balance, interest paid, lease payment and closing balance. Use the spreadsheet to answer the following questions. a) What part of the $305 is used to pay the interest on the $7105? b) What total price is being charged for the car? c) What is the change in the monthly lease payment, if the lease-end value is reduced to $5700? d) What is the monthly payment for a straight purchase over 36 months with a 20% down payment? e) What is the annual percentage depreciation rate assumed with the $7105 lease-end value?

Strand: Number (Number Operations)

Students will:

- demonstrate an understanding of and proficiency with calculations
- decide which arithmetic operation or operations can be used to solve a problem and then solve the problem.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[E]	Estimation and Mental Mathematics	**[T]**	Technology
		[V]	Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
(continued)	A8–4. (N23) Use spreadsheet(s) to analyze an investment or life insurance portfolio, applying such concepts as capital gains, interest rate, inflation rate, risk, total rate of return and after-tax rate of return. [C, PS, T]	4.1 The time needed for an investment to double in value can be estimated using the rule of 72, which states that $n = \frac{72}{i}$ where i is the annual percentage interest rate and n the number of years. a) Compare the rule of 72 doubling time with the exact doubling time for the following interest rates: • 4% per annum, compounded annually • 8% per annum, compounded annually • 24% per annum, compounded annually. b) What general conclusion can be drawn as to the accuracy of rule of 72 calculations? 4.2 An average car in 1996 costs $20 000. a) If this money were invested for 15 years at 8% per year, compounded yearly, and cars did not increase in price, how many cars could be bought in 2011? b) If the average inflation rate were 3.5% per year, how many cars could be bought in 2011 with the proceeds from the investment? c) What is the real, after inflation, rate of return for the investment? d) How do the answers change, if 40% of the interest is taken in income tax every year? 4.3 A retirement portfolio of $300 000 is to be invested for a 10-year period. A middle-risk stock has a probability of 0.80 of making a 110% capital gain and paying annual dividends of 3.2%; there is a 0.20 probability of making a 30% capital loss and paying no annual dividends. Term deposits are guaranteed to pay interest at 7.5% per year, compounded annually. a) What is the best net worth, if all the capital is invested in stocks and the stocks make the maximum capital gain? b) What is the worst net worth, if all the capital is invested in stocks and the stocks take the maximum capital loss? c) Compare the expected net worth from the stocks to the guaranteed net worth from the term deposits. d) How would the numbers in the problem be different for high-risk stocks and for low-risk stocks? e) Modify the calculations to allow for 40% of the gains to be paid yearly in income tax.

Strand: Number (Number Operations)

Students will:

- demonstrate an understanding of and proficiency with calculations
- decide which arithmetic operation or operations can be used to solve a problem and then solve the problem.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

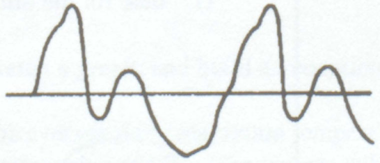
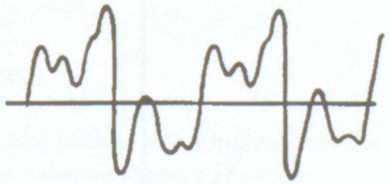
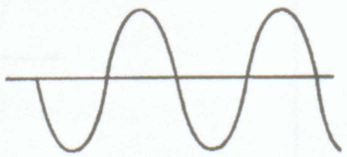
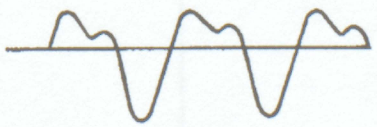
General Outcomes	Specific Outcomes	Illustrative Examples
(continued)	A8–5. (N24) Analyze car or house insurance needs and premiums, using such concepts as loss, probability of loss, compulsory coverage, optional coverage, deductible and claims record. [CN, E, R, T]	5.1 Obtain collision damage figures for inexperienced drivers and for experienced drivers from an insurance company, and then calculate a fair insurance premium for $1 000 000 liability, $250 deductible collision and $100 deductible comprehensive theft/glass coverage. Do the calculation twice, once for each type of driver. What change in premium would be fair, if the deductible for collision were raised to $1000? 5.2 At what point is it worth it to drop collision coverage on an older vehicle? Show a strategy, and explain the supporting calculations. 5.3 How long does a home security system need to be installed before the cost of the system is paid for by the savings in insurance premiums? Obtain data for your area from an insurance agent. Show a strategy, and explain the supporting calculations.

Strand: Patterns and Relations (Patterns)

Students will:

- use patterns to describe the world and to solve problems.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[E]	Estimation and Mental Mathematics	**[T]**	Technology
		[V]	Visualization

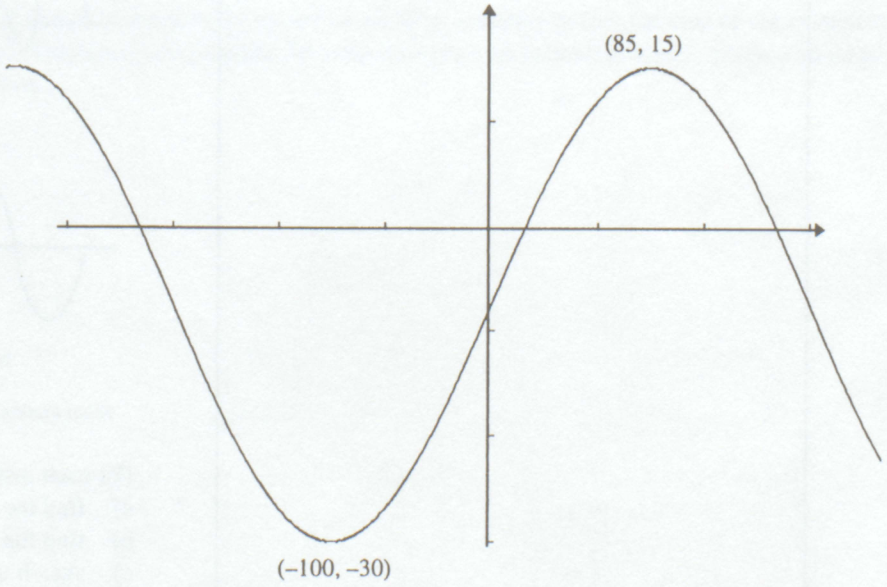
General Outcomes	Specific Outcomes	Illustrative Examples
Generate and analyze cyclic, recursive and fractal patterns.	A7–1. (PR10) From cyclic data produce a periodic graph. [C, PS, V]	1.1 Research the sunrise time for a period of one year, and graph it. From your graph, determine the time of sunrise for March 12.
(continued)	A7–2. (PR11) Predict results from graphs that represent periodic events. [E, R, V]	2.1 The following are graphs showing the patterns produced on an oscilloscope when four different musical instruments are played.  violin  clarinet  tuning fork  organ pipe From *Fundmentals of Physics* by Martindale et al. Reprinted by permission of ITP Nelson Canada. For each instrument: a) find the amplitude b) find the period c) sketch the graph, if the instrument is played louder d) sketch the graph, if the instrument is used to play a higher note.

Strand: Patterns and Relations (Patterns)

Students will:

- use patterns to describe the world and to solve problems.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[E]	Estimation and Mental Mathematics	**[T]**	Technology
		[V]	Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
(continued)	A7–3. (PR12) Describe periodic events, including sinusoidal curves, using correct terminology. [C, V]	3.1 A temperature–time graph was drawn for a northern Saskatchewan town. The variable plotted on the horizontal axis is the calendar date, with April 1 as zero and the unit being days. The variable plotted on the vertical axis is the temperature in degrees Celsius. The graph is drawn below. Find the: a) amplitude b) period c) maximum and minimum values d) vertical shift e) date for the maximum temperature f) date for the minimum temperature. 

Strand: Patterns and Relations (Patterns)

Students will:

- use patterns to describe the world and to solve problems.

[C] Communication
[CN] Connections
[E] Estimation and Mental Mathematics
[PS] Problem Solving
[R] Reasoning
[T] Technology
[V] Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
(continued)	A7–4. (PR13) Collect sinusoidal data; sketch the graph of the data; and, using degrees, represent the data with an equation of the form: • $y = a \sin (kt) + c$ **OR** • $y = a \cos (kt) + c$. [CN, PS, T, V]	4.1 Collect data from real-world situations, such as: a) hours of daylight b) low tide and high tide c) average low and average high temperatures on different dates of the year. Plot the data, and determine an approximate equation for the data in the form of: $y = a \sin (kt) + c$ **or** $y = a \cos (kt) + c$.
	A7–5. (PR14) Develop sinusoidal equations, using degrees, to represent periodic behaviour. [CN, PS, T]	5.1 Sketch a graph, and build an equation to represent the following situation. The average daily maximum temperature in Vancouver follows a sinusoidal pattern with a highest value of 24°C and a lowest value of 8°C. The highest value occurs on July 15 and the lowest value on January 15.

Strand: Patterns and Relations (Patterns)

Students will:

- use patterns to describe the world and to solve problems.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

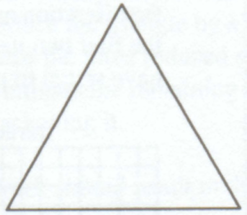
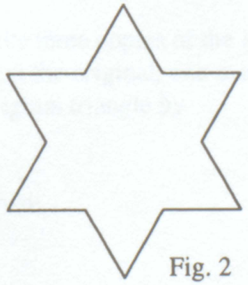
General Outcomes	Specific Outcomes	Illustrative Examples
(continued)	A7–6. (PR15) Use technology to generate and graph finite or infinite sequences whose recursive definition may or may not be given. [PS, T, V]	6.1 For the Fibonacci sequence 1, 1, 2, 3, 5, . . . , determine a recursive form. 6.2 Find the 20th term of the sequence $t_n = t_{n-1} + 2$, where $t_1 = 1$, by generating a table or graph. 6.3 A sequence is defined by $t_n = 3t_{n-1} + 2t_{n-2}$. Determine the value of t_9, given $t_0 = 5$ and $t_1 = 3$. Use a spreadsheet to find t_{100} and the first term of the sequence that has a value of more than 1 million.
	A7–7. (PR16) Identify sequences that appear to be: • divergent • convergent • oscillating • static. [C, V]	7.1 Calculate several terms of the following sequences where the n^{th} term is defined as follows: a) $a_n = 6^{n+1}$ b) $a_n = (-2)^n$ c) $a_n = 6$ d) $a_n = \frac{1}{2n}$. Graph the results. Use this information to hypothesize each of the sequences as divergent, convergent, oscillating or static. 7.2 The monthly closing balances of a loan form a sequence. Under what conditions will these balances form a divergent sequence? 7.3 Regular polygons of n sides are inscribed in a circle of radius 10 cm. The perimeters P_n of these regular polygons form a sequence. Is this sequence convergent? Estimate the value of P_n, if n is very large.

Strand: Patterns and Relations (Patterns)

Students will:

- use patterns to describe the world and to solve problems.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
(continued)	A7–8. (PR17) Construct a fractal pattern by repeatedly applying a procedure to a geometric figure. [CN, R, V]	8.1 The following example is the Koch snowflake curve. Construct an equilateral triangle (Fig. 1). Trisect each side, construct an equilateral triangle on each middle third, and delete the middle third (Fig. 2).  Fig. 1  Fig. 2 For each segment in Fig. 2, repeat the above. 8.2 Construct your own fractal pattern.
	A7–9. (PR18) Use the concept of self-similarity to compare and/or predict the perimeters, areas and volumes of fractal patterns. [CN, R, V]	9.1 For illustrative example 8.1, predict the perimeter of the fifth pattern.
	(continued)	

Strand: Patterns and Relations (Patterns)

Students will:

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[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
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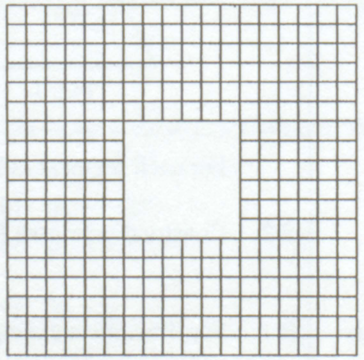
		[V]	Visualization

<table>
<tr><th>General Outcomes</th><th>Specific Outcomes</th><th>Illustrative Examples</th></tr>
<tr><td>(continued)</td><td>(continued)</td><td>

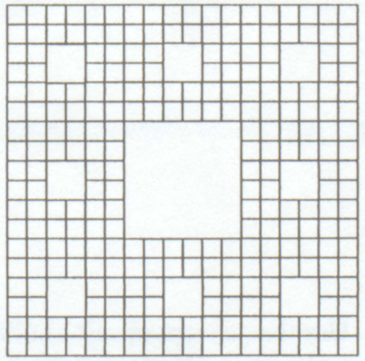
9.2 Fractal Carpet

A fractal can be generated by a pattern of iteration. This fractal design is called the Sierpinski carpet after the mathematician who invented it in 1916. The general rule is to start with a square and take a square out. Look at the first iteration and describe the rule that was used to determine the size of the square that was removed. Now compare the first two iterations and describe the rule that was used to construct the second from the first. Apply the rule you have stated to construct the third iteration in the space provided.

Iteration 1



Iteration 2



Iteration 3

Now examine the third iteration you have constructed, and record the length of the side of the new squares you drew. Compare this length to the lengths of the sides of the previous squares. Write the lengths of the sides of all the squares in descending order. If you construct the fourth iteration, what will the lengths of the sides of the squares need to be? Now look at the first iteration again. What is the area of the square that was removed? What is the area of each individual square that was removed in the next two iterations? Write these areas in descending order. What is the area of each individual square to be removed in the fourth iteration?

Challenge: Find the perimeter of all the squares in the third iteration. Find the area of the figure that remains once all the squares are removed in the third iteration.

Excerpted and adapted with permission from *Geometry from Multiple Perspectives* (*Curriculum and Evaluation Standards* Addenda Series, Grades 9–12), copyright 1991 by the National Council of Teachers of Mathematics. All rights reserved.

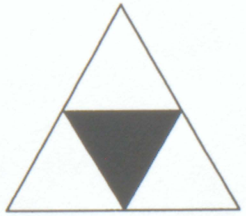
</td></tr>
</table>

Strand: Patterns and Relations (Patterns)

Students will:

- use patterns to describe the world and to solve problems.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
(continued)	*(continued)*	9.3 The Sierpinski triangle can be created by using dilations and isometries. You may begin with an arbitrary triangle. An equilateral triangle is used for the procedures described below. a) Draw an equilateral triangle. b) Reduce the triangle by a factor of ½. Make three copies of the reduced triangle. c) Place the three reduced similar triangles on the original, one at each vertex. d) Eliminate the remaining portion of the original triangle by blackening it.  Your work should result in the figure shown here. Answer the following questions: a) Let the area of the original triangle be 1 area unit. What area remains? What area has been removed? b) Let the side of the original triangle be 1 length unit. What is the perimeter of the figure with the dark region removed? Repeat steps a) through d) of the original procedure for each of the triangular regions remaining in the figure shown. Sketch the result of your work. Answer the following questions: a) What is the area of the remaining triangular region? b) What is the perimeter of the new "holey" triangular region? c) What would the next iteration of the procedure look like? Make a sketch. d) Write an expression for the area of the Sierpinski triangle after carrying out the procedure *n* times. e) Write an expression for the perimeter of the Sierpinski triangle after carrying out the procedure *n* times. f) How would your expressions differ, if you began with a triangle other than an equilateral triangle? Excerpted and adapted with permission from *Geometry from Multiple Perspectives* (*Curriculum and Evaluation Standards* Addenda Series, Grades 9–12), copyright 1991 by the National Council of Teachers of Mathematics. All rights reserved.

Strand: Patterns and Relations (Patterns)

Students will:

- use patterns to describe the world and to solve problems.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

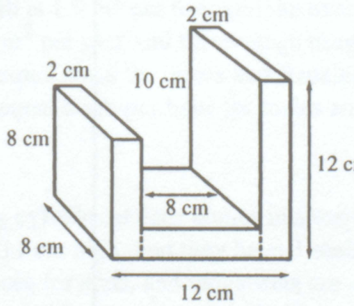
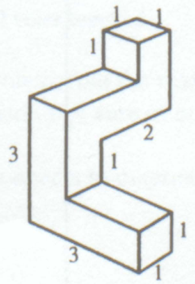
General Outcomes	Specific Outcomes	Illustrative Examples
(continued)	*(continued)*	9.4 Construct a cylinder with the dimensions: $r = 10$ cm, $h = 20$ cm. A second figure is constructed by halving the previous radius and height. A third is constructed by halving the second and so on. a) Predict the surface area and the volume of the sixth pattern. b) Write an expression for the surface area after carrying out the procedure n times. c) Write an expression for the volume after carrying out the procedure n times.

Strand: Shape and Space (Measurement)

Students will:

- describe and compare everyday phenomena, using either direct or indirect measurement.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

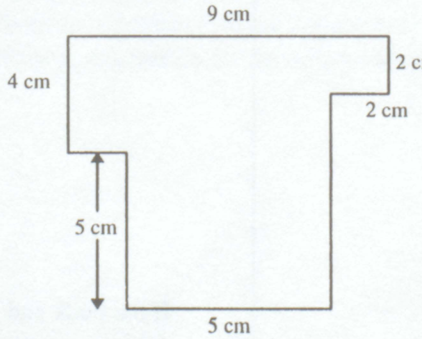
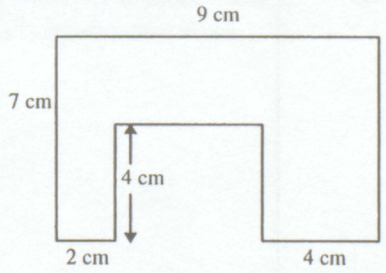
General Outcomes	Specific Outcomes	Illustrative Examples
Analyze objects, shapes and processes to solve cost and design problems.	A9–1. (SS15) Use dimensions and unit prices to solve problems involving perimeter, area and volume. [E, PS, V]	1.1 Determine the volume of the plastic book end shown below.  2 cm 2 cm 10 cm 8 cm 12 cm 8 cm 8 cm 12 cm If the book end is constructed using an injection mold, find the development cost if the plastic ingredients cost 6¢ per cubic centimetre. 1.2 In the following diagram of an outside storage system component, all the angles are right angles and the lengths are in centimetres. Find the volume. A special aluminum latex coating is applied to all outside surfaces of the object. What is the cost of the latex coating, if it costs 28¢ per cm^2? 
(continued)	*(continued)*	

Strand: Shape and Space (Measurement)

Students will:

- describe and compare everyday phenomena, using either direct or indirect measurement.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[E]	Estimation and Mental Mathematics	**[T]**	Technology
		[V]	Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
(continued)	*(continued)*	1.3 A dressmaker cuts pairs of the following shapes from a rectangular piece of gabardine that is 1 m by 0.5 m. Determine the maximum number of pairs that can be cut from the piece of gabardine. Identify any assumptions.  
	A9–2. (SS16) Solve problems involving estimation and costing for objects, shapes or processes when a design is given. [C, E, PS]	2.1 A swimming pool is 50 m by 21 m. The deep end is 4.0 m deep and extends out 12 m. The shallow end is 1.2 m deep and extends out 12 m. There is a uniform slope connecting the deep and shallow ends. a) Draw scale diagrams showing the top view and the side view of the pool. b) Calculate the cost of filling it with water at $\$2.00/\text{m}^3$. c) Waterproofing of the underwater surfaces costs $\$17/\text{m}^2$. Determine the cost of waterproofing.
		2.2 A window cleaner has been asked by the owner of a large office tower to submit a quotation for cleaning the windows of the building. The window cleaner has the following information: a) there are 24 floors b) there are 14 windows per side on each floor c) there are 4 sides to the building. From experience, the window cleaner knows that the transfer time between windows on the same floor and same side of the building is 60 seconds. The transfer time between sides of the building is 120 seconds and between floors is 30 seconds. The time to clean one window is 120 seconds. The window cleaner has a base charge of $120. The maximum period of time he works at one stretch is 3 hours, then he takes a 30 minute rest. In addition to his rate of $25/hour, he wants to make 25% profit from the job for reinvestment in his business. What would be the best quote?
	(continued)	

Strand: Shape and Space (Measurement)

Students will:

- describe and compare everyday phenomena, using either direct or indirect measurement.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

<table>
<tr><th>General Outcomes</th><th>Specific Outcomes</th><th>Illustrative Examples</th></tr>
<tr><td><i>(continued)</i></td><td><i>(continued)</i></td><td>2.3 To satisfy the building code, an auditorium has to have 1200 m^2 of washroom space. In a washroom for males, the average space needed is 1.9 m^2 per user and the average usage time is 97 s. In a washroom for females, the average space needed is 2.4 m^2 per user and the average usage time is 145 s. Determine the required washroom space:
a) on the basis of equal areas for males and females
b) on the basis of equal users per hour for males and females.</td></tr>
<tr><td></td><td>A9–3. (SS17) Design an object, shape, layout or process within a specified budget. [PS, R, V]

<i>(continued)</i></td><td>3.1 Tin plate for making cylindrical cans comes in sheets that are 240 cm by 160 cm and costs $3.20 per sheet. Cans are 6 cm in diameter and 11 cm high, and they have 3 seals each. Seals cost 0.8¢ each to make. One sheet of tin plate is used for making pieces for ends, and two sheets are used for making pieces for sides.
a) How many ends and how many sides can be made from the three sheets of tin plate?
b) How many cans can be made from the three sheets, and what is the cost per can?
c) Is there another way of making more cans from the three sheets, or the same number of cans from less tin plate?
d) How much money is saved doing it the second way?

3.2 To produce a voters' list for a riding, a sum of $1.70 per voter is allocated. Four methods of enumerating are possible:
<table>
<tr><th>Method</th><th>Cost per Voter</th><th>Probability of Return</th></tr>
<tr><td>Hand deliver enumeration form, mail return</td><td>$0.91</td><td>0.700</td></tr>
<tr><td>Mail form both ways</td><td>$1.07</td><td>0.740</td></tr>
<tr><td>Telephone until voter reached</td><td>$2.21</td><td>0.920</td></tr>
<tr><td>Enumerator calls until voter reached</td><td>$5.26</td><td>0.995</td></tr>
</table>
For a total of 40 000 voters, find the maximum number of voters who can be enumerated within the budget and the minimum budget needed to be sure of enumerating 98% of the potential voters.

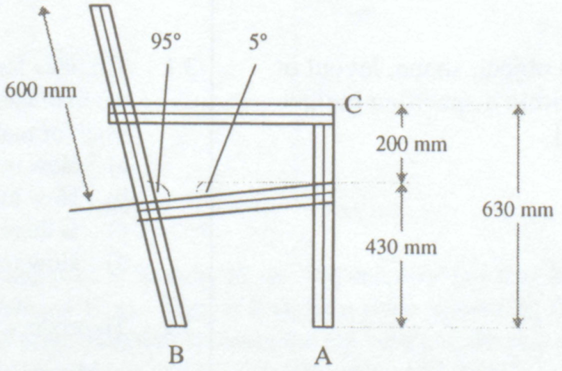
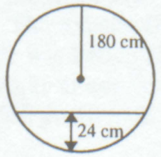
Note: This problem connects to outcomes in clusters A5 and C6.</td></tr>
</table>

Strand: Shape and Space (Measurement)

Students will:

- describe and compare everyday phenomena, using either direct or indirect measurement.

[C] Communication
[CN] Connections
[E] Estimation and Mental Mathematics
[PS] Problem Solving
[R] Reasoning
[T] Technology
[V] Visualization

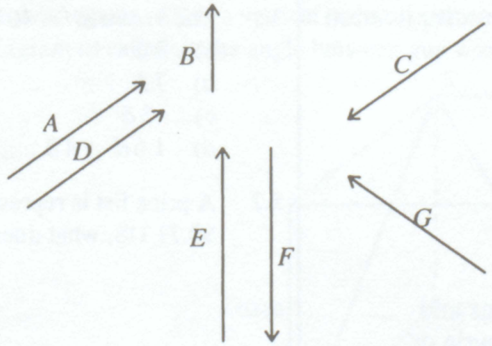
General Outcomes	Specific Outcomes	Illustrative Examples
(continued)	*(continued)*	3.3 One side of a wooden chair is being built. The front of the seat should be 430 mm above the ground and should slope back at 5° from the horizontal. The seat depth is 450 mm, and the angle between the seat and the back of the chair is 95°. The required length of the back of the chair, measured from the seat, is 600 mm. The height of the horizontal chair arm is 200 mm above the front of the seat. Draw a scale diagram, and use it to calculate the lengths of wooden components A, B and C. What is the maximum cost per metre for the wood needed to make this side of the chair, if the cost cannot exceed $20?  95° 5° 600 mm C 200 mm 630 mm 430 mm B A
	A9–4. (SS18) Use simplified models to estimate the solutions to complex measurement problems. [E, V]	4.1 Estimate the area of the Yukon Territory, by: a) counting squares b) splitting the area into rectangles and triangles. Which method is most accurate? Which type of map gives the most reliable estimate for the area of the Yukon Territory? Where are the main sources of error in the estimate? 4.2 A water tank is a sphere of diameter 3.6 m. Estimate the volume of water in the tank, if the depth of water is 24 cm. 

Strand: Shape and Space (3-D Objects and 2-D Shapes)

Students will:

- describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

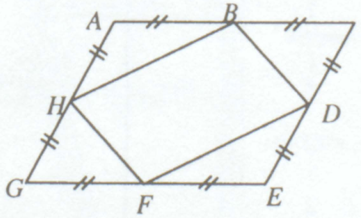
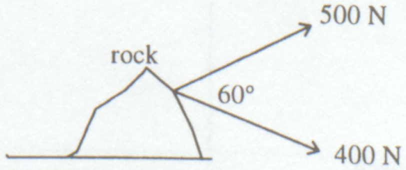
General Outcomes	Specific Outcomes	Illustrative Examples
Solve problems involving polygons and vectors, including both 3-D and 2-D applications. *(continued)*	A6–4. (SS30) Use and give 3-D and 2-D examples of vector terminology and notation, including: • vector (direction, magnitude) • scalar • unit vector • collinear vectors • opposite vectors • parallel vectors • resultant vectors. [C, CN]	4.1  Given the above vectors, complete the following chart. opposite vectors parallel vectors resultant vectors collinear vectors 4.2 Car *A* is travelling at 110 km/h and Car *B* is travelling at 100 km/h. a) Give an example where the magnitude of $A - B$ is equal to 210 km/h. b) Give an example where the magnitude of $A - B$ is equal to 10 km/h. c) If *A* and *B* are at right angles, what is the magnitude of $A - B$?

Strand: Shape and Space (3-D Objects and 2-D Shapes)

Students will:

- describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

[C] Communication
[CN] Connections
[E] Estimation and Mental Mathematics
[PS] Problem Solving
[R] Reasoning
[T] Technology
[V] Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
(continued)	A6–5. (SS31) Assign meaning to the multiplication of a vector by a scalar. [CN]	5.1 The vector $\vec{a}$ is 40 km/h east. Make a scale drawing of each of the following vectors: a) $3\vec{a}$ b) $7\vec{a}$ c) $-3\vec{a}$ d) $1.6\vec{a} + 4\vec{a}$. 5.2 A price list is represented in Canadian dollars by the vector $\vec{p} = (27, 38, 14, 26)$. If the Canadian dollar is worth \$0.71 US, what does the vector $\vec{q} = 0.71\vec{p}$ represent?
	A6–6. (SS32) Perform vector additions and subtractions, using triangle or parallelogram methods. [V]	6.1  Using the above diagram of a rhombus *ACEG*, determine the vector addition of each of the following: a) $AH + HG$ b) $GF + BC$ c) $GF + CB$ d) $FD + DE$. 6.2 A ski jumper encounters a horizontal friction of 85 N backward, a vertical weight of 750 N downward and an air resistance of 340 N upward. Draw the vector addition of these forces, and use the drawing to find the magnitude and direction of the resultant force.
	A6–7. (SS33) Determine the magnitude and direction of a resultant vector, using triangle, parallelogram or component methods. [CN, T, V]	7.1 A boat is travelling across a river with a forward velocity of 14 m/s, and there is a current of 3 m/s down the river. How fast is the boat travelling? 7.2 John and Marie are using two ropes to pull a rock. Draw a vector diagram to estimate the magnitude and direction of the resultant force. Verify the estimate by a calculation, using components. 

[redacted]

Strand: Shape and Space (3-D Objects and 2-D Shapes)

Students will:

- describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[E]	Estimation and Mental Mathematics	**[T]**	Technology
		[V]	Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
(continued)	*(continued)*	8.4 Model, by drawing a diagram, Jack's jogging route, if he jogs north at 15 km/h for 30 minutes and then turns east and jogs at 12 km/h for 20 minutes. How far has he jogged in total? How far is he from his starting point? In what direction does he need to go to return to the start by the shortest path?

Strand: Statistics and Probability (Chance and Uncertainty)

Students will:

- use experimental or theoretical probability to represent and solve problems involving uncertainty.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[E]	Estimation and Mental Mathematics	**[T]**	Technology
		[V]	Visualization

<table>
<tr><th>General Outcomes</th><th>Specific Outcomes</th><th>Illustrative Examples</th></tr>
<tr>
<td rowspan="2">Use normal and binomial probability distributions to solve problems involving uncertainty.

(continued)</td>
<td>C6–1. (SP11) Find the population standard deviation of a data set or a probability distribution, using technology.
[CN, E, T, V]</td>
<td>1.1 Measure the height of each student in a class, and calculate the mean and standard deviation.

1.2 A company uses an automated packaging device to produce 50-g bags of Karmel Korn. The machine needs frequent checking to see if it is actually putting 50 g in each bag. The following are the masses, in grams, of thirty bags of Karmel Korn.
<table>
<tr><td>54</td><td>50</td><td>47</td><td>50</td><td>51</td><td>50</td></tr>
<tr><td>53</td><td>50</td><td>47</td><td>51</td><td>50</td><td>51</td></tr>
<tr><td>52</td><td>49</td><td>46</td><td>52</td><td>50</td><td>49</td></tr>
<tr><td>52</td><td>48</td><td>48</td><td>53</td><td>49</td><td>49</td></tr>
<tr><td>51</td><td>48</td><td>49</td><td>52</td><td>49</td><td>50</td></tr>
</table>
a) Calculate the mean and standard deviation of this data.
b) What problems will be encountered, if the standard deviation gets too high?

Dottori et al., <i>Foundations of Mathematics 11</i>, p. 392. Adapted with permission.</td>
</tr>
<tr>
<td>C6–2. (SP12) Use <i>z</i>-scores and <i>z</i>-score tables to solve problems.
[PS, R, T, V]

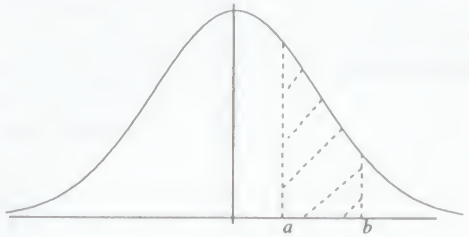
(continued)</td>
<td>2.1 The volume of the contents of a soft drink can is normally distributed about a mean of 350 mL, with a standard deviation of 1.5 mL.
a) Calculate the <i>z</i>-score for a can with a volume of 355 mL.
b) What percentage of production will consist of cans having content volumes between 350 mL and 355 mL?
c) What percentage of production will consist of cans having content volumes less than 355 mL?
d) If cans containing less than 346 mL must be rejected, how many cans will be expected to be rejected in a run of 50 000?</td>
</tr>
</table>

Strand: Statistics and Probability (Chance and Uncertainty)

Students will:

- use experimental or theoretical probability to represent and solve problems involving uncertainty.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[E]	Estimation and Mental Mathematics	**[T]**	Technology
		[V]	Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
(continued)	*(continued)*	2.2  a) What is the area under this curve? b) If $P(a < z < b) = 0.4$, what is the area under the curve for the interval $a < z < b$? c) If $P(z < b) = 0.9$, calculate $P(z > b)$, and calculate the value of b. 2.3 For entry into the Canadian Armed Forces, the standards for height used to be set at 158 cm to 194 cm for males, and 152 cm to 184 cm for females. Use the concept of z-score to test if these two height standards are equivalent. Assume means of 176 cm and 163 cm and standard deviations of 8 cm and 7 cm respectively. 2.4 A sample of 122 people gives a mean body temperature of 36.8°C, with a standard deviation of 0.35°C. Assuming a normal distribution, find: a) the expected number of people with temperatures above 37.0°C b) the expected number of people with temperatures below 36.0°C. Also, estimate the range of temperatures contained within the sample. 2.5 In the general population, the IQ scores of individuals is normally distributed with a mean of 110 and a standard deviation of 10. If a large group of people is tested: a) What proportion of this group is expected to have IQs between 100 and 120? b) What is the probability that an individual in the group has an IQ greater than 120?

Strand: Statistics and Probability (Chance and Uncertainty)

Students will:

- use experimental or theoretical probability to represent and solve problems involving uncertainty.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

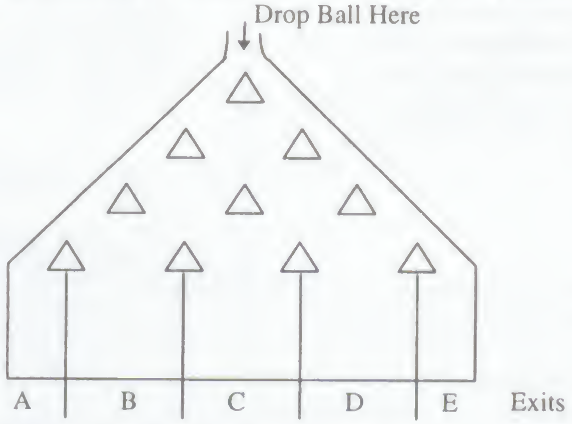
General Outcomes	Specific Outcomes	Illustrative Examples
(continued)	C6–3. (SP13) Use the normal distribution and the normal approximation to the binomial distribution to solve problems involving confidence intervals for large samples. [CN, E, PS]	3.1 The heights of males employed by a manufacturer follow a normal distribution with a mean of 169 cm and a standard deviation of 8 cm. a) Establish a symmetric 95% confidence interval for the average height in a random sample of 36 male employees. b) What happens to the width of the symmetric 95% confidence interval, if the sample size is increased from 36 to 225? 3.2 Pollsters estimate that the number of decided voters in favour of a particular bylaw is 64%, and the number opposed is 36%. a) If the sample size is 250, find the expected mean and standard deviation of *yes* voters. b) Estimate, for this sample, the expected percentage of *yes* voters, with a symmetric 95% confidence interval used to establish the margin of error. c) If the margin of error for the percentage of *yes* voters must be less than ±1.0%, what would be the minimum sample size required? 3.3 The probability that a car salesperson will complete a sale is 0.10. If the salesperson has 200 customers in the next month, establish a symmetric 95% confidence interval for the number of completed sales for the month.

Strand: Statistics and Probability (Chance and Uncertainty)

Students will:

- use experimental or theoretical probability to represent and solve problems involving uncertainty.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[E]	Estimation and Mental Mathematics	**[T]**	Technology
		[V]	Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
Solve problems based on the counting of sets, using techniques such as the fundamental counting principle, permutations and combinations.	C6–4. (SP14) Solve pathway problems, interpreting and applying any constraints. [PS, R]	4.1 Given the following "pinball" situation, what is the probability of the ball reaching each of the exits?  What assumptions are made in the solution?
	C6–5. (SP15) Use the fundamental counting principle to determine the number of different ways to perform multistep operations. [PS, R]	5.1 Joe has three different shirts, two different pairs of pants and five different pairs of shoes. List all possible outfits in such a way as to ensure that all have been counted and none have been counted twice. How many possible outfits are there? Use the fundamental counting principle to determine the number of outfits there should be. Do your answers match? 5.2 An airline pilot reported that in seven days she spent one day in Winnipeg, one day in Regina, two days in Edmonton and three days in Yellowknife. How many different itineraries are possible? What difference would it make if the first day and the last day had to be spent in Yellowknife?

Strand: Statistics and Probability (Chance and Uncertainty)

Students will:

- use experimental or theoretical probability to represent and solve problems involving uncertainty.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[E]	Estimation and Mental Mathematics	**[T]**	Technology
		[V]	Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
Model the probability of a compound event, and solve problems based on the combining of simpler probabilities.	C6–6. (SP20) Construct a sample space for two or three events. [PS, R, V]	6.1 List the sample space for rolling a 6-sided die and flipping a coin. 6.2 Draw or list the sample space for the following situation. A bus is scheduled to arrive at a train station at any time between 07:05 and 07:15 inclusive. A train is scheduled to arrive between 07:11 and 07:17 inclusive. The arrival of a bus at 07:06 and a train at 07:14 can be represented by the point (6, 14). Times are expressed in whole minutes. a) How many points are there in this sample space? b) How many points have the bus and the train arriving at the same time? c) How many points have the bus arriving after the train? d) What is the probability of the bus arriving after the train?
	C6–7. (SP21) Classify events as independent or dependent. [C]	7.1 Classify the following events as independent or dependent: a) tossing a head in a coin toss and rolling a 6 on a die b) drawing an ace for the first card and another ace for the second, if the experiment is carried out without replacement c) drawing a king for the first card and a queen for the second, if the experiment is carried out with replacement. 7.2 Sixty per cent of young drivers take driver training, and 25% of young drivers have an accident in their first year of driving. Statistics show that 10% of those who do take driver training have an accident in their first year. Are taking driver training and having an accident in the first year independent events?
	C6–8. (SP22) Solve problems, using the probabilities of mutually exclusive and complementary events. [CN, PS, R]	8.1 If the probability of winning a game is $\frac{1}{31}$, what is the probability of losing the game? 8.2 A shootout consists of teams A and B taking alternate shots on goal. The first team to score wins. Team A has a probability of 0.3 of scoring with any one shot. Team B has a probability of 0.4 of scoring with any one shot. a) If Team A shoots first, what is the probability of Team B winning on its first shot? b) If Team A shoots first, what is the probability of Team A winning on its third shot?

Strand: Number (Number Concepts)

Students will:

- use numbers to describe quantities
- represent numbers in multiple ways.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[E]	Estimation and Mental Mathematics	**[T]**	Technology
		[V]	Visualization

General Outcomes

Analyze the numerical data in a table for trends, patterns and interrelationships.

(continued)

Specific Outcomes

C1–1. (N1) Use words and algebraic expressions to describe the data and the interrelationships in a table with rows that are not related recursively (not calculated from previous data). [C, CN]

Illustrative Examples

1.1

Price	GST	PST	Total
$120.00	$ 8.40	$12.84	$141.24
$275.00	$19.25	$29.43	$323.68

a) What is the rate of GST?
b) What could be the rate of PST?
c) What could be the rule for calculating PST?
d) What is the total GST paid on the two items in the table?
e) What is the total PST paid on the two items in the table?

1.2 National Hockey League (NHL)
Western Conference: February 1, 1996

	W	L	T	Points
Detroit	35	9	4	74
Colorado	26	14	9	61
Chicago	25	15	11	61
Toronto	22	19	9	53
St. Louis	21	20	8	50
Winnipeg	21	24	4	46
Vancouver	17	20	12	46
Los Angeles	17	22	11	45
Calgary	18	23	9	45
Edmonton	18	25	6	42
Anaheim	17	27	5	39
Dallas	14	24	10	38
San Jose	11	35	4	26

What happens to the NHL standings if wins are worth three points and ties are worth one point?

Strand: Number (Number Concepts)

Students will:

- use numbers to describe quantities
- represent numbers in multiple ways.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[E]	Estimation and Mental Mathematics	**[T]**	Technology
		[V]	Visualization

General Outcomes

(continued)

Specific Outcomes

C1–2. (N2) Use words and algebraic expressions to describe the data and the interrelationships in a table with rows that are related recursively (calculated from previous data). [C, CN]

Illustrative Examples

2.1 The following table provides data on the repayment of a $100 000 farm loan. The farmer has negotiated for one annual payment to be made each year after harvest and for the right to make an extra payment, if the harvest is good. Use the table to answer the questions.

Year	Opening Balance	Interest Rate (%)	Interest Charged	Regular Payment	Extra Payment	Closing Balance
1	$100 000.00	8	$8000.00	$14 902.95		$93 097.05
2	$ 93 097.05	8	$7447.76	$14 902.95		$85 641.87
3	$ 85 641.87	8	$6851.35	$14 902.95		$77 590.27
4	$ 77 590.27	8	$6207.22	$14 902.95		$68 894.54
5	$ 68 894.54	8	$5511.56	$14 902.95		$59 503.15
6	$ 59 503.15	8	$4760.25	$14 902.95		$49 360.46
7	$ 49 360.46	8	$3948.84	$14 902.95		$38 406.34
8	$ 38 406.34	8	$3072.51	$14 902.95		$26 575.90
9	$ 26 575.90	8	$2126.07	$14 902.95		$13 799.03
10	$ 13 799.03	8	$1103.92	$14 902.95		$ 0.00

a) What is the period of the loan?
b) What is the amount of the annual payment?
c) How much of the annual payment at the end of Year 5 went toward the opening balance? Show how to determine the answer in two different ways.
d) Create an algebraic expression to find the answer in c).
e) If the interest rate went up to 11% in Year 10, how much would be owing at the end of Year 10?
f) What extra payment at the end of Year 4 would pay the loan off at the end of Year 8?

Strand: Number (Number Concepts)

Students will:

- use numbers to describe quantities
- represent numbers in multiple ways.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[E]	Estimation and Mental Mathematics	**[T]**	Technology
		[V]	Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
Explain and illustrate the structure and the interrelationship of the sets of numbers within the real number system.	C1–3. (N3) Classify numbers as natural, whole, integer, rational or irrational, and show that these number sets are nested within the real number system. [C, R, V]	3.1 Explain why the number 1.112111211112 . . . is irrational. 3.2 Given a set of numbers, place them in their appropriate box in a nested Venn diagram. 3.3 Describe, orally and in writing, whether or not a number is irrational. 3.4 Demonstrate that a particular real number, such as $\sqrt{3}$, is rational or irrational.
	C1–4. (N4) Use approximate representations of irrational numbers. [R, T]	4.1 Compare the results of using different approximations for $\sqrt{2}$ in calculations. a) Calculate $\sqrt{2} \times \sqrt{2}$ as 1.4×1.4. b) Calculate $\sqrt{2} \times \sqrt{2}$ as 1.41×1.41. 4.2 Use a calculator to get the approximate value, to four decimal places, of $\sqrt{8}$ and of $2\sqrt{2}$.

Strand: Number (Number Operations)

Students will:

- demonstrate an understanding of and proficiency with calculations
- decide which arithmetic operation or operations can be used to solve a problem and then solve the problem.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[E]	Estimation and Mental Mathematics	**[T]**	Technology
		[V]	Visualization

<table>
<tr><th>General Outcomes</th><th>Specific Outcomes</th><th>Illustrative Examples</th></tr>
<tr>
<td>Describe and apply arithmetic operations on tables to solve problems, using technology as required.

<i>(continued)</i></td>
<td>C1–7. (N7) Create and modify tables from both recursive and nonrecursive situations. [PS, T, V]</td>
<td>7.1
<table>
<tr><th>Price</th><th>GST</th><th>PST</th><th>Total</th></tr>
<tr><td>$120.00</td><td>$ 8.40</td><td>$12.84</td><td>$141.24</td></tr>
<tr><td>$275.00</td><td>$19.25</td><td>$29.43</td><td>$323.68</td></tr>
</table>
a) Modify the table to allow for a PST of 6.5% of the price before taxes.
b) If the price after both taxes is $138.00 and PST is charged on the $120.00 price before taxes, what is the rate of PST?

7.2 In 1993, sales of a particular video game doubled every month. The game was released in May 1993 with sales of 32 000 for May. Prepare a table to illustrate the 1993 monthly sales figures. How many video games were sold in December 1993? Identify the assumptions you made when determining the solution.
In 1994, the demand for the video game peaked. Starting in January 1994, and every month thereafter, sales were cut to one quarter of what they were in the previous month. How many video games were sold in April 1994? If April 1994 was the last month of sales, how many video games were sold over the entire twelve months?</td>
</tr>
</table>

Strand: Number (Number Operations)

Students will:

- demonstrate an understanding of and proficiency with calculations
- decide which arithmetic operation or operations can be used to solve a problem and then solve the problem.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[E]	Estimation and Mental Mathematics	**[T]**	Technology
		[V]	Visualization

<table>
<tr><th>General Outcomes</th><th>Specific Outcomes</th><th>Illustrative Examples</th></tr>
<tr>
<td><i>(continued)</i></td>
<td>C1–8. (N8) Use and modify a spreadsheet template to model recursive situations.
[PS, T, V]</td>
<td>
8.1 Modify the given template for a 10-year, $85 000 farm mortgage with fixed annual payments, to allow for a change in interest rate.
<table>
<tr><th>Year</th><th>Opening Balance</th><th>Interest Rate (%)</th><th>Interest Charged</th><th>Regular Payment</th><th>Closing Balance</th></tr>
<tr><td>1</td><td>$85 000.00</td><td>8</td><td>$6800.00</td><td>$12 667.51</td><td>$79 132.49</td></tr>
<tr><td>2</td><td>$79 132.49</td><td>8</td><td>$6330.60</td><td>$12 667.51</td><td>$72 795.59</td></tr>
<tr><td>3</td><td>$72 795.59</td><td>8</td><td>$5823.65</td><td>$12 667.51</td><td>$65 951.73</td></tr>
<tr><td>4</td><td>$65 951.73</td><td>8</td><td>$5276.14</td><td>$12 667.51</td><td>$58 560.36</td></tr>
<tr><td>5</td><td>$58 560.36</td><td>8</td><td>$4684.83</td><td>$12 667.51</td><td>$50 577.68</td></tr>
<tr><td>6</td><td>$50 577.68</td><td>8</td><td>$4046.21</td><td>$12 667.51</td><td>$41 956.39</td></tr>
<tr><td>7</td><td>$41 956.39</td><td>8</td><td>$3356.51</td><td>$12 667.51</td><td>$32 645.39</td></tr>
<tr><td>8</td><td>$32 645.39</td><td>8</td><td>$2611.63</td><td>$12 667.51</td><td>$22 589.52</td></tr>
<tr><td>9</td><td>$22 589.52</td><td>8</td><td>$1807.16</td><td>$12 667.51</td><td>$11 729.17</td></tr>
<tr><td>10</td><td>$11 729.17</td><td>8</td><td>$ 938.33</td><td>$12 667.51</td><td>$ 0.00</td></tr>
</table>
a) What alternatives are open to the farmer, if the interest rate increases?
b) What alternatives are open to the farmer, if the interest rate decreases?

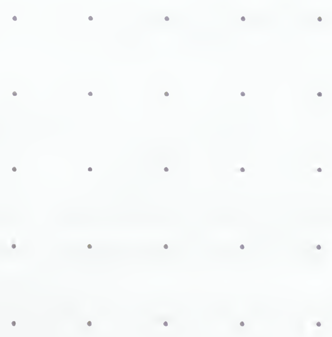
8.2 Modify the template in illustrative example 8.1 to reflect a 25-year home mortgage with monthly payments that gives the customer the option of making an annual extra payment of $1500 at the end of any year. Interest is charged monthly.
</td>
</tr>
</table>

Strand: Number (Number Operations)

Students will:

- demonstrate an understanding of and proficiency with calculations
- decide which arithmetic operation or operations can be used to solve a problem and then solve the problem.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
Use exact values, arithmetic operations and algebraic operations on real numbers to solve problems.	P1–1. (N10) Explain and apply the exponent laws for powers of numbers and for variables with rational exponents. [C, E]	1.1 Find the exact value of $\left(\frac{8}{27}\right)^{\left(-\frac{2}{3}\right)}$ 1.2 Write the number expression $7^{\left(\frac{2}{3}\right)}$, using radicals. 1.3 Simplify $\left(\sqrt[5]{x^3}\right)\left(\sqrt[3]{x^2}\right)$. 1.4 Show $\left(\sqrt[3]{-8}\right)x = -2x$. 1.5 Write an equivalent expression for $\sqrt[3]{\sqrt[2]{3x^5}}$, using exponents. 1.6 Prove that $\sqrt{2}$ is an irrational number. 1.7 The 5 × 5 geoboard shown in the diagram can be used to construct squares whose areas are whole numbers. The sides of the squares can be constructed by joining dots horizontally, vertically or diagonally. What whole number areas can be constructed? Justify your answers with appropriate drawings and calculations. 

Strand: Number (Number Operations)

Students will:

- demonstrate an understanding of and proficiency with calculations
- decide which arithmetic operation or operations can be used to solve a problem and then solve the problem.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[E]	Estimation and Mental Mathematics	**[T]**	Technology
		[V]	Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
Use exact values, arithmetic operations and algebraic operations on real numbers to solve problems.	P2–1. (N11) Perform operations on irrational numbers of monomial and binomial form, using exact values. [E]	1.1 Show that $\sqrt{2}+\sqrt{8}=3\sqrt{2}$. 1.2 Find an equivalent form of $\left(\frac{3}{\sqrt{5}-\sqrt{2}}\right)$ that has a whole number as its denominator. 1.3 Arrange the following in order from least to greatest: 7, $2\sqrt{13}$, $3\sqrt{6}$, $4\sqrt{5}$, $5\sqrt{2}$. Do not use decimal approximations. 1.4 Find the exact value of $\sqrt[3]{128}+4\left(\sqrt[3]{16}\right)$. 1.5 Find an equivalent form of $\left(3\sqrt{5}+4\sqrt{2}\right)\left(4\sqrt{5}-3\sqrt{2}\right)$. 1.6 An equilateral triangle is inscribed in a circle. If the area of the circle is 36π, find the exact area of the equilateral triangle.

Strand: Patterns and Relations (Patterns)

Students will:

- use patterns to describe the world and to solve problems.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
Generate and analyze number patterns.	P2–2. (PR1) Generate number patterns exhibiting arithmetic growth. [E, R]	2.1 The first modern Olympiad was held in 1896. Every four years after this date the summer Olympics were held. Given such a framework, reveal what should have been the next five summer Olympic years after 1896. Explain why this pattern was never achieved. 2.2 The output of a northern gold mine has remained constant at 2200 ounces per year. If, at the end of last year, the total output of the mine was 122 600 ounces of gold, what will be the total output at the end of this year? At the end of next year? 2.3 A salesperson receives a base salary of $12 000 per year, plus $100 for every unit sold. What is the salary, if 50 units are sold? 51 units? 52 units? 2.4 For the arithmetic sequence 16, 23, 30, 37, . . . , find the next three terms. 2.5 A pile of bricks is arranged in rows. The numbers of bricks in the rows form an arithmetic sequence. There are 45 bricks in the 5th row and 33 bricks in the 11th row. a) How many bricks are in the first row? b) Write the general term for the sequence. c) What is the maximum number of rows of bricks possible?
(continued)	P2–3. (PR2) Use expressions to represent general terms and sums for arithmetic growth, and apply these expressions to solve problems. [CN, PS, R, T]	3.1 For the arithmetic sequence 7, 11, 15, 19, . . . , find the 29th term. 3.2 Find the sum of the arithmetic series 3 + 7 + 11 + . . . + 483. 3.3 Mary's annual salary is on a range from $26 785 in the first year to $34 825 in the seventh year. a) If the salary range is an arithmetic sequence with seven terms, determine the raise Mary can expect each year. b) What is her salary in the fifth year? c) What is the first salary in this range that is greater than $30 000? d) What is the total amount that Mary earned in the seven years?

Strand: Patterns and Relations (Patterns)

Students will:

- use patterns to describe the world and to solve problems.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
(continued)	P2–4. (PR3) Relate arithmetic sequences to linear functions defined over the natural numbers. [CN]	4.1 If three eggs are used for every carrot cake made in a bakery, write a function that determines the number of eggs for *n* cakes. 4.2 To rent an ice arena, there is an initial charge for cleaning the ice, plus a rental fee for each hour or part of an hour. The rates posted on the board are:
		(see table below) Graph the function that models the rates posted on the board.
	P2–5. (PR4) Generate number patterns exhibiting geometric growth. [E, R]	5.1 Insert three numbers between 5 and 80, so that the five numbers form a geometric sequence. 5.2 A store is conducting a Dutch auction. It will take 10% off the cost of an item each day. If an item originally costs $150, find its cost for each of the next five days.

Time (h)	Cost ($)
Less than 1	100
More than 1, less than 2	180
More than 2, less than 3	260
. . .	

Strand: Patterns and Relations (Variables and Equations)

Students will:

- represent algebraic expressions in multiple ways.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[E]	Estimation and Mental Mathematics	**[T]**	Technology
		[V]	Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
Generalize operations on polynomials to include rational expressions.	P1–2. (PR22) Factor polynomial expressions of the form $ax^2 + bx + c$, and $a^2x^2 - b^2y^2$. [E]	2.1 Factor: a) $5x^2 + 6x - 8$ b) $6x^2 - x - 2$. 2.2 Factor $4x^2 + 20x + 25$. a) Compare the two factors. b) For this special product, what is the relationship between the coefficients of the terms of the factors and the coefficients of the terms of the trinomial? 2.3 Factor $4x^2 - 25$. a) Compare the two factors. b) For this special product, what is the relationship between the coefficients of the terms of the factors and the coefficients of the terms of the binomial? 2.4 For which integral values of k can $4x^2 + kx + 3$ be factored over the set of rational numbers? 2.5 Factor $(x + b)^2 + 6(x + b) + 8$. 2.6 Factor $6x^4 - x^2 - 2$.
(continued)	P1–3. (PR23) Find the product of polynomials. [E, R]	3.1 Find the product and simplify: a) $(3x - 4)(2x^2 + 3x + 1)$ b) $(2x - y)^3$.

Strand: Patterns and Relations (Variables and Equations)

Students will:

- represent algebraic expressions in multiple ways.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
(continued)	P1–4. (PR24) Divide a polynomial by a binomial, and express the result in the forms: • $\frac{P}{D}=Q+\frac{R}{D}$ • $P = DQ + R$ • $P(x) = D(x)Q(x) + R$. [E, R]	4.1 Divide $(3x^3 + 2x^2 - 7x + 8)$ by $(x + 2)$. 4.2 Divide $(t^2 - 3t - 10)$ by $(t - 3)$. 4.3 Divide $(6x^3 - 2x^2 + 7x - 11)$ by $(3x^2 - 2)$. 4.4 When the polynomial $P(t) = 4t^4 - 17t^2 - 36t - 20$ is divided by $(2t - 5)$, the remainder is -60. Express the division in the forms: a) $\frac{P(t)}{2t-5}=Q(t)+\frac{R}{2t-5}$ b) $P(t) = Q(t)\,(2t - 5) + R$.
	P1–5. (PR25) Determine equivalent forms of simple rational expressions with polynomial numerators, and denominators that are monomials, binomials or trinomials that can be factored. [PS, R]	5.1 Change each rational expression to its simplest equivalent form: a) $\frac{4x^4-6x^3+2x^2-10x}{2x}$ b) $\frac{x^2-5x-6}{x^2-36}$ c) $\frac{x^2+3x}{x^2+x-6}$ d) $\frac{16x^4-81y^4}{(4x^2+9y^2)^2\,(2x^2-xy-3y^2)}$.

Strand: Patterns and Relations (Variables and Equations)

Students will:

- represent algebraic expressions in multiple ways.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[E]	Estimation and Mental Mathematics	**[T]**	Technology
		[V]	Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
(continued)	P1–6. (PR26) Determine the nonpermissible values for the variable in rational expressions. [C, CN]	6.1 For what value(s) of x are each of the following not defined? Explain your conclusion in each case. a) $\frac{3}{x}$ b) $\frac{-2}{x+1}$ c) $\frac{4}{3x-4}$ d) $\frac{2x+1}{x^2-4}$ e) $\frac{5x}{x^2-3x-4}$ f) $\frac{5x+y}{3x-y}$ g) $\frac{7x^2-6xy+3y^2}{4x^2-9y^2}$ h) $\frac{2}{x^3}$ i) $\frac{5}{\left(x^3-1\right)}$

Strand: Patterns and Relations (Variables and Equations)

Students will:

- represent algebraic expressions in multiple ways.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[E]	Estimation and Mental Mathematics	**[T]**	Technology
		[V]	Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
(continued)	P1–8. (PR28) Find and verify the solutions of rational equations. [CN, PS]	8.1 Solve for x, checking for any nonpermissible values. a) $\frac{2}{x}=-3$ b) $\frac{4}{x}+\frac{3}{2x}=\frac{11}{4}$ c) $\frac{5}{x-1}-\frac{2}{x+1}=2$ d) $\frac{2x+1}{x+3}-\frac{x-2}{x+1}=5$ e) $\frac{3}{x^2-25}+\frac{2}{x+5}=\frac{4}{x-5}$ f) $\frac{4}{x-5}+6=\frac{4}{x-5}$ 8.2 The average speed of an airplane is five times as fast as the average speed of a passenger train. To travel 400 km, the train requires 4 hours more than the airplane. Find the average speeds of the train and the airplane.

Strand: Patterns and Relations (Relations and Functions)

Students will:

- use patterns to describe the world and to solve problems.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[E]	Estimation and Mental Mathematics	**[T]**	Technology
		[V]	Visualization

<table>
<tr><th>General Outcomes</th><th>Specific Outcomes</th><th>Illustrative Examples</th></tr>
<tr>
<td>Examine the nature of relations with an emphasis on functions.</td>
<td>C1–9. (PR47) Plot linear and nonlinear data, using appropriate scales.
[C, V]</td>
<td>
9.1 The mass of a beaker is recorded when the beaker contains varying volumes of ethanol. The results of the experiment are recorded in the table below.
<table>
<tr><th>Volume of Ethanol (mL)</th><th>Mass of Beaker and Liquid (g)</th></tr>
<tr><td>0</td><td>90</td></tr>
<tr><td>50</td><td>129</td></tr>
<tr><td>100</td><td>168</td></tr>
<tr><td>150</td><td>207</td></tr>
<tr><td>200</td><td>246</td></tr>
</table>
Measurements may be assumed correct to the nearest mL and to the nearest g.

Plot this data on a scatterplot, using appropriate scales, and answer the following questions.
a) Assuming that this pattern continues, determine the mass of the beaker and liquid when 250 mL of ethanol is present.
b) When a volume of 200 mL of ethanol is in the beaker, determine the mass of the ethanol alone.
c) The density of a liquid is defined as the mass of 1 mL of the liquid. Determine the density of the ethanol.

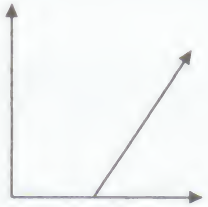
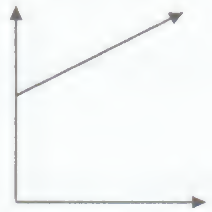
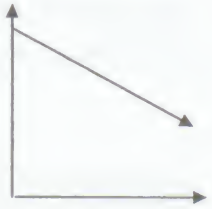
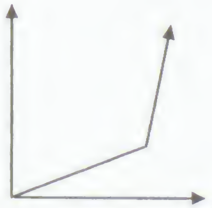
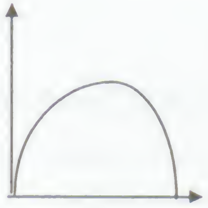
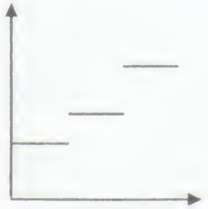
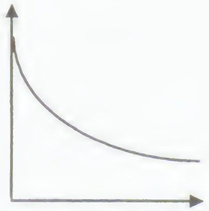
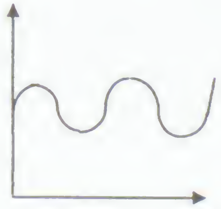
9.2 Nannook's Pizza uses the following price structure.
<table>
<tr><th>Diameter (inches)</th><th>Cost ($)</th></tr>
<tr><td>8</td><td>6.50</td></tr>
<tr><td>10</td><td>10.20</td></tr>
<tr><td>12</td><td>14.65</td></tr>
<tr><td>14</td><td>19.90</td></tr>
<tr><td>16</td><td>26.00</td></tr>
</table>
Plot this data on a scatterplot, using appropriate scales, and describe the pattern.
</td>
</tr>
</table>

Strand: Patterns and Relations (Relations and Functions)

Students will:

- use algebraic and graphical models to generalize patterns, make predictions and solve problems.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[E]	Estimation and Mental Mathematics	**[T]**	Technology
		[V]	Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
Examine the nature of relations with an emphasis on functions. *(continued)*	C2–1. (PR48) Represent data, using function models. [CN, PS, V]	1.1 Sketch graphs to illustrate the following situations. If sufficient information is given, represent the situation by a suitable equation. Sketch and, if possible, represent by an equation: a) the area of a circle as a function of its radius b) the cost of mailing a letter as a function of the mass of the letter c) the cost of renting a car for one day as a function of the kilometres driven d) the population of Canada as a function of the year e) the length of daylight as a function of the date. 1.2 For each of the following graphs, describe a practical situation that could be represented by the graph. In describing the situation, state the meanings of any intercepts, slopes, maxima and/or minima.  

Strand: Patterns and Relations (Relations and Functions)

Students will:

- use algebraic and graphical models to generalize patterns, make predictions and solve problems.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[E]	Estimation and Mental Mathematics	**[T]**	Technology
		[V]	Visualization

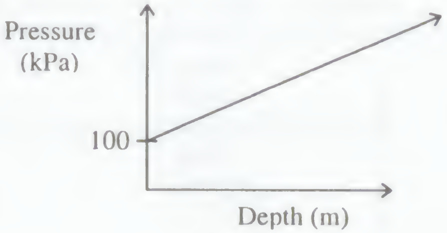
General Outcomes	Specific Outcomes	Illustrative Examples
(continued)	C2–6. (PR53) Determine the following characteristics of the graph of a linear function, given its equation: • intercepts • slope • domain • range. [PS, V]	6.1 A tanker truck drives on a weigh scale and then is filled with crude oil. The mass M, measured in kilograms, of the truck and the volume V, measured in barrels, of crude oil are related by the formula: $M = 14\ 000 + 180\ V;\ V \leq 500.$ a) Draw the graph with V on the horizontal axis and M on the vertical axis. b) The tank has a maximum capacity of 500 barrels. What is the mass of the truck when it contains 500 barrels of oil? c) What is the mass of the empty truck? Where is this value found on the graph? d) Find the slope, and give an interpretation for it. e) Give the domain for this problem. f) Express the range in words. 6.2 Graph each of the following equations; and indicate intercepts, slope, domain and range. a) $y = 2x;\ x = (0, 1, 2, 3, 4, 5, 6)$ b) $y = -\frac{1}{3}x;\ x =$ a real number c) $y = 3$ d) $x = 3$ e) $y = \frac{1}{3}x + 5;\ x =$ a real number f) $y = mx + b;\ x =$ a real number

Strand: Patterns and Relations (Relations and Functions)

Students will:

- use algebraic and graphical models to generalize patterns, make predictions and solve problems.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[E]	Estimation and Mental Mathematics	**[T]**	Technology
		[V]	Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
Represent data, using linear function models. *(continued)*	C2–7. (PR56) Use direct variation and arithmetic sequences as applications of linear functions. [CN, PS, V] *(continued)*	7.1 A hydrologist studied the relationship between the pressure on an object and its depth of submersion in a liquid. The following graph was sketched. Draw conclusions based upon the sketch.  7.2 Simple interest varies directly with the amount borrowed. a) If the interest is $5 for $100 borrowed, what would the interest be for $325 borrowed? b) Graph the relation, and write the equation of the graph. 7.3 A jet ski rental operation at Lake Okanagan charges a fixed insurance premium, plus an hourly rate. The total cost for two hours is $50 and for five hours is $110. a) Graph the relation. b) Determine the fixed insurance premium and the hourly rate to rent the jet ski. 7.4 With new equipment coming on line, a soft drink manufacturer has been increasing its production each day according to the following table. Assume a maximum daily output of 25 000 cans. Day 1 2 3 4 Units 4000 4200 4400 4600 a) Graph the relation. Hint: this is a discrete case. b) On what day will they be able to produce 20 000 cans, if this trend continues?

Strand: Shape and Space (Measurement)

Students will:

- describe and compare everyday phenomena, using either direct or indirect measurement.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[E]	Estimation and Mental Mathematics	**[T]**	Technology
		[V]	Visualization

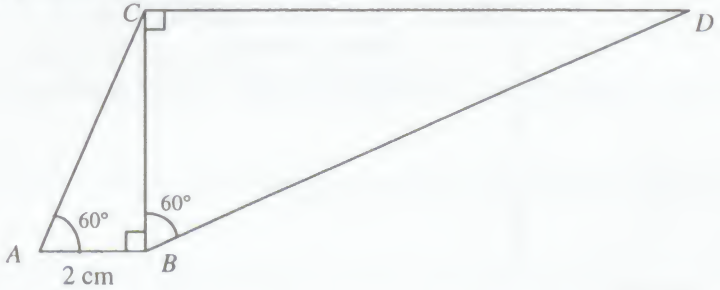
General Outcomes	Specific Outcomes	Illustrative Examples
Demonstrate an understanding of scale factors, and their interrelationship with the dimensions of similar shapes and objects.	C3–1. (SS1) Calculate the volume and surface area of a sphere, using formulas that are provided. [CN, PS, V]	1.1 Calculate the volume and surface area of a beach ball of radius 15 cm. 1.2 A hot air balloon has a spherical shape and a diameter of 4 m. If 30 additional cubic metres of air are pumped into the balloon, what will be the new values for the diameter, volume and surface area?
	C3–2. (SS2) Determine the relationships among linear scale factors, areas, the surface areas and the volumes of similar figures and objects. [CN, PS, R, V]	2.1 The area of a region in a plane is 10 cm^2. By what factor must each of the dimensions of this region be multiplied to increase the area by 20 cm^2? 2.2 A model train is built to a scale of 1:50. If the length of the model engine is 20 cm and the area of sheet metal used to cover the outside surface of the model is 180 cm^2, what is the actual length of the engine and the actual area of the sheeting used to cover the engine? If the volume displaced by the model engine is 126 cm^3, what is the volume displaced by the real engine, in m^3? 2.3 It is improbable that a giant human, 6 m in height (three or four times normal human height), could exist. Which biological systems are most likely to break down? Explain your answer.

Strand: Shape and Space (Measurement)

Students will:

- describe and compare everyday phenomena, using either direct or indirect measurement.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[E]	Estimation and Mental Mathematics	**[T]**	Technology
		[V]	Visualization

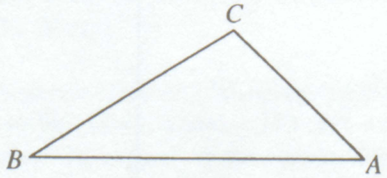
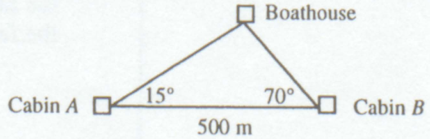
General Outcomes	Specific Outcomes	Illustrative Examples
Solve problems involving triangles, including those found in 3-D and 2-D applications.	C3–3. (SS4) Solve problems involving two right triangles. [CN, PS, V]	3.1 From the top of a 100 m fire tower, a fire ranger observes two fires, one at an angle of depression of 5° and the other at an angle of depression of 2°. Assuming that the fires and the tower are in a straight line, determine the distance between the fires for the following: a) when the fires are on the same side of the tower b) when the fires are on opposite sides of the tower. 3.2 The triangles *ABC* and *BCD* have right angles at *B* and *C* respectively. Calculate the length of side *CD*, and state the ratio of length *BD* to length *AC*.  3.3 Canada's highest waterfall is Della Falls on Vancouver Island. An observer standing at the same level as the base of the falls views the top of the falls at an angle of elevation of 58°. When the observer moves 31 m closer to the base of the falls, the angle of elevation increases to 61°. Find the height of Della Falls.
	C3–4. (SS5) Extend the concepts of sine and cosine for angles from 0° to 180°. [R, T, V]	4.1 Find sin 130°. 4.2 Use a calculator to find multiple solutions for angle *A*, if sin *A* = sin 130°. Use trial and error to find as many solutions as possible. Summarize the pattern found in the solutions.
(continued)	*(continued)*	

Strand: Shape and Space (Measurement)

Students will:

- describe and compare everyday phenomena, using either direct or indirect measurement.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[E]	Estimation and Mental Mathematics	**[T]**	Technology
		[V]	Visualization

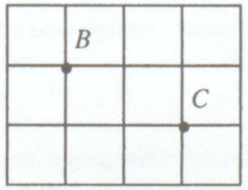
General Outcomes	Specific Outcomes	Illustrative Examples
(continued)	*(continued)*	4.3 Find the value(s) for A ($0° \le A \le 180°$) when $\sin A = \frac{1}{2}$. Find the value(s) for A ($0° \le A \le 180°$) when $\cos A = \frac{1}{2}$. Find the value(s) for A ($0° \le A \le 180°$) when $\cos A = -\frac{1}{2}$.
	C3–5. (SS6) Apply the sine and cosine laws, excluding the ambiguous case, to solve problems. [CN, PS, V]	5.1 An electric transmission line is planned to go directly over a pond. The power line will be supported by posts at points A and B. A surveyor measures the distance from B to C as 580 m, the distance from A to C as 337 m and $\angle BCA$ as 105.34°. What is the distance from post A to post B? 
		5.2 Two cabins are located 500 m apart on the same side of a river. Across the river from the two cabins is a boathouse. This situation is illustrated in the diagram below. Use the measurements to find the width of the river. 
		5.3 A farmer has a field in the shape of a triangle. From one corner, it is 530 m to the second corner and 750 m to the third corner. The angle between the lines of sight to the second and to the third corners is 53°. Find the perimeter and area of the field.
		5.4 A sailboat leaves the dock at Gibson's Landing on a bearing of S57°W. After sailing for 8 km, the ship tacks and travels S31°E for 5 km. a) How far is the sailboat from Gibson's Landing? b) What direction would it have to sail to return to the dock at Gibson's Landing? Bye et al., *Holtmath 11*, p. 313. Reprinted with permission.

Strand: Shape and Space (3-D Objects and 2-D Shapes)

Students will:

- describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[E] Estimation and Mental Mathematics	[T] Technology
	[V] Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
Solve coordinate geometry problems involving lines and line segments.	C1–10. Solve problems involving distances between points in the coordinate plane. (SS19) [PS, V]	10.1 Bob and Christine want to meet; see map below. Each block has dimensions of 120 m by 120 m. Assuming the roads are of negligible width, how far does Bob *B* have to travel to get to Christine *C*? Find two separate answers, one for a path along the roads and one for a direct path.  10.2 Plot the points (–4, –2) and (1, 5) on the coordinate plane. Describe two different ways to calculate the distance between the two points. 10.3 Generate a method of determining the distance between any two points in the coordinate plane without having to plot the points. Justify your method. 10.4 Program a calculator or computer to accept, as input, the coordinates of two points and to give, as output, the distance between the two points. Document the program so that someone else can use it without assistance.
	C1–11. Solve problems involving midpoints of line segments. (SS20) [PS]	11.1 Explain to a partner the meaning of the midpoint of the line segment joining two points without using the word midpoint. 11.2 On a map with numerical coordinates in kilometres, the village of Sundown is at (6.3, 2.9), while the town of Sunup is at (4.7, 13.2). It was decided to construct a water main on the direct line joining Sunup with Sundown. Each community was responsible for the cost of construction from the community to the midpoint. Find the coordinates of the midpoint and Sundown's costs, if Sundown spent \$63 475 per kilometre for construction. Determine alternative methods that could be used to solve the problem.
(continued)	C1–12. Solve problems involving rise, run and slope of line segments. (SS21) [PS, V]	12.1 If the slope of a line is 6 ($m = 6$) and the line passes through the points (2, 5) and (1, k), what is the value of k? 12.2 If two points on a line are (4, 3) and (6, 4), find one other point on the line. Use a graphing utility to demonstrate the reasonableness of your answer.

Strand: Shape and Space (3-D Objects and 2-D Shapes)

Students will:

- describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

[C] Communication
[CN] Connections
[E] Estimation and Mental Mathematics
[PS] Problem Solving
[R] Reasoning
[T] Technology
[V] Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
(continued)	C1–13. Determine the equation of a line, given information that uniquely determines the line. (SS22) [PS, V]	13.1 Use a graphing device to examine changes in the graph of $y = mx + b$ as the values of m and b are changed. Use the results to explain why the equation $y = mx + b$ is called the slope and y-intercept form of a linear equation. 13.2 Write a clear explanation of the nature of the following lines: $x = a$, $y = b$, $x = y$. 13.3 Manipulate the standard form of a straight line ($Ax + By + C = 0$) into the slope and y-intercept form of the same line. Determine rules that connect A, B and C to the slope (m) and to the intercepts. 13.4 Find the equation of a line passing through the points $(-1, 3)$ and $(4, 2)$. 13.5 Given the graph of an oblique line, determine an equation for the line. 13.6 A spring with no masses attached is 25.2 cm long. For each 1-g mass attached to the spring, the spring's length increases by 4 mm. Graph this scenario, label the axes, and find an equation for the graph.
	C1–14. Solve problems using slopes of: (SS23) • parallel lines • perpendicular lines. [CN, PS, V]	14.1 Graphically examine the slopes of various lines, all of which are perpendicular to the line $y = \frac{2}{3}x + 2$. Describe the slopes, and make a rule for finding the slope of a perpendicular to a given line. 14.2 Two perpendicular lines intersect on the x-axis. The equation of one of the lines is $y = 2x - 6$. Find the equation of the second line.

Strand: Statistics and Probability (Data Analysis)

Students will:

- collect, display and analyze data to make predictions about a population.

[C] Communication
[CN] Connections
[E] Estimation and Mental Mathematics
[PS] Problem Solving
[R] Reasoning
[T] Technology
[V] Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
Implement and analyze sampling procedures, and draw appropriate inferences from the data collected.	C3–6. (SP1) Choose, justify and apply sampling techniques that will result in an appropriate, unbiased sample from a given population. [C, PS, R]	6.1 A toothpaste company advertises that three out of four dentists prefer their product. Analyze this statement for its completeness and its accuracy in terms of population, sample, possible sampling technique, validity and bias. 6.2 A school cafeteria wants to introduce a new dessert. Describe how a survey could be conducted to decide which of three choices should be the new dessert. 6.3 To predict a winner in a federal election, a magazine compiled a list of about 200 000 names from sources, such as telephone books, lists of automobile owners, club membership lists and its own subscription lists. The magazine mailed a questionnaire to everybody on the list, and 4000 returned it. The 4000 responses became the sample. Discuss the potential sources of bias.
	C3–7. (SP2) Defend or oppose inferences and generalizations about populations, based on data from samples. [C, PS, R]	7.1 To determine a preference for spending $50 in either a clothing store, an electronics shop or a restaurant, customers were surveyed one Saturday morning at the mall. Fifty-nine per cent preferred spending in a clothing store, 32% in an electronics shop and 9% in a restaurant. What generalizations can be made from these results? Does the sample adequately represent the population to be surveyed? Design a more reliable sampling method to obtain this information, and include details of the questionnaires used and the method of selecting the sample. 7.2 Search through various forms of media to find examples of generalizations that have been made about populations, based on data from samples. Do you agree or disagree with the generalizations? Explain why.

Strand: Statistics and Probability (Chance and Uncertainty)

Students will:

- use experimental or theoretical probability to represent and solve problems involving uncertainty.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[E]	Estimation and Mental Mathematics	**[T]**	Technology
		[V]	Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
Make and analyze decisions, using expected gains and losses, based on the probabilities of simple events. *(continued)*	P2–6. (SP9) Connect probabilities to calculated expected gains or losses. [CN, PS, R, V]	6.1 A business person is preparing a proposal for a computer contract worth $12 000. This person estimates that it would cost $1500 to prepare the proposal, and the probability of receiving the contract is estimated to be 0.20. Find this business person's expected gain. 6.2 The Khan family is considering moving from Calgary to Hamilton. In Calgary, Ali earns $46 000 and Kareema earns $34 000. Based on the family's research, if they move, Ali has an estimated probability of 0.85 of finding a job that pays $53 000, and an estimated probability of 0.12 of finding a job that pays $33 000. Otherwise he would be unemployed, receiving $17 000. Kareema has an estimated probability of 0.65 of finding a job that pays $62 000, and an estimated probability of 0.12 of finding a job that pays $33 000. Otherwise she would be unemployed, receiving $11 000. What is the expected gain in salary, if the Khans move to Hamilton? 6.3 Sherry takes a 100-item multiple-choice examination. Each item has four possible choices. She knows 68 of the answers and guesses randomly at the other 32. Calculate her expected number of correct answers.

Strand: Statistics and Probability (Chance and Uncertainty)

Students will:

- use experimental or theoretical probability to represent and solve problems involving uncertainty.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[E]	Estimation and Mental Mathematics	**[T]**	Technology
		[V]	Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
(continued)	P2–7. (SP10) Solve decision-making problems involving expected values, and communicate the solutions. [C, PS, R]	7.1 Dave and Tony are playing toss-up with two coins. Dave wins one point, if both coins are heads or both are tails. Tony wins one point, if the two coins are different. After 100 tosses, what are the two players' expected scores? Is this a fair game? 7.2 Joe paid $5 to throw a pair of dice. He wins the sum of the numbers appearing on the top faces of the dice, unless a six appears on either die; then he wins nothing. a) Is this a fair game? b) What difference would it make if the six were changed to a one? c) Justify your answers by analyzing the sample space for this dice throw. 7.3 Obtain collision damage figures for inexperienced drivers and for experienced drivers from an insurance company, and then calculate a fair insurance premium for $1 000 000 liability, $250 deductible collision and $100 deductible comprehensive theft/glass coverage. Do the calculation twice, once for each type of driver. What change in premium would be fair, if the deductible for collision were raised to $1000? 7.4 At what point is it worth it to drop collision coverage on an older vehicle? Show a strategy, and explain the supporting calculations. 7.5 Explain why it is reasonable to insure a house against fire damage, where the probability of collecting is 0.003, but it is not reasonable for a bank, using current interest rates, to make a loan that has a 90% probability of getting repaid. 7.6 The growing of grapes for *Eiswein* involves harvesting the grapes as late as possible in October. As each day passes, the grapes become more valuable, but there is a greater risk of a frost killing the grapes and reducing their value. For a particular year, the value of the grape juice is $2.00/L on October 1, and the value of the juice increases by $0.15/L per day for every day in October. The probability of a killer frost is 0.03 for any particular day in October. After a killer frost, the value of the juice is $1.50/L. On what day does the risk of frost damage outweigh the gain from extra maturing time?

MATHEMATICS 11

derived from

The Common Curriculum Framework

for

K–12 MATHEMATICS

Grade 10 to Grade 12

Western Canadian Protocol for Collaboration in Basic Education

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JUNE 1996

Strand: Number (Number Operations)

Students will:

- demonstrate an understanding of and proficiency with calculations
- decide which arithmetic operation or operations can be used to solve a problem and then solve the problem.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[E]	Estimation and Mental Mathematics	**[T]**	Technology
		[V]	Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
Solve consumer problems, using arithmetic operations. *(continued)*	C4–1. (N12) Solve consumer problems, including: • wages earned in various situations • property taxation • exchange rates • unit prices. [CN, E, PS, R, T]	1.1 Calculate and compare wage situations involving minimum wage rates, regular pay, overtime pay, gratuities, piecework, straight commission, salary and commission, salary plus quota and graduated commission. 1.2 Jane has a choice of two restaurants at which to work. Mario's pays $8/h, and tips average $24 daily. Teppan's pays $5.50/h, and tips average $35 daily. If Jane works 30 hours weekly, spread over four days, how much would she earn at each restaurant? 1.3 Identify and calculate various payroll deductions, including income tax, CPP, UI, medical benefits, union and professional dues and life insurance premiums. 1.4 Estimate, calculate and compare gross and net pay for various wage or salary earners in your community. 1.5 The Ningart property has a market value of $105 000. The assessed values in the area are 60% of market values. The tax rate is 32.3 mills of assessed value. What is the Ningarts' monthly tax payment? 1.6 The exchange rate on a given day in the United States is 28% and in Canada 38.8%. Explain why this is possible. 1.7 A Canadian traveller goes from Switzerland to Germany. She knows that one Swiss franc is equivalent to $1.26 Canadian (including exchange cost) and that one German mark is $0.97 Canadian (including exchange cost). How many German marks does she get for 100 Swiss francs? 1.8 Which provides better value for tomato soup, $0.69 for 284 mL or $1.79 for 907 mL?

Strand: Number (Number Operations)

Students will:

- demonstrate an understanding of and proficiency with calculations
- decide which arithmetic operation or operations can be used to solve a problem and then solve the problem.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General Outcomes

(continued)

Specific Outcomes

C4–3. (N14) Solve budget problems, using graphs and tables to communicate solutions. [C, PS, T, V]

C4–4. (N15) Plot and describe data of exponential form, using appropriate scales. [C, T, V]

Illustrative Examples

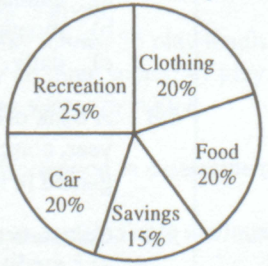
3.1 Research and calculate the cost of running a car for a year. Decide how to classify each cost, how to collect the data and how to display the results.

3.2 As a project, prepare a monthly budget for one of the following:

a) the family
b) an assumed persona; e.g., Wayne Gretzky
c) a school
d) a vacation
e) a fishing/hunting/shopping trip
f) a municipality.

3.3 The diagram shows Julie's monthly budget of $1200. She wants to move to her own apartment that costs $450 per month. Construct a new budget that will include her rent. Explain the choices and changes that Julie could make.

Julie Barnes' Monthly Budget



Total = $1200

4.1 The growth of the value of a $7000 RRSP is as follows:

Time (years)	Value ($)
0	7 000
1	7 630
2	8 316
3	9 065
4	9 881
5	10 770

Plot this data, estimate the time needed for the RRSP to reach $14 000, and determine the value of the RRSP after 12 years.

(continued)

Strand: Number (Number Operations)

Students will:

- demonstrate an understanding of and proficiency with calculations
- decide which arithmetic operation or operations can be used to solve a problem and then solve the problem.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General Outcomes

(continued)

Specific Outcomes

C4–2. (N13) Reconcile financial statements including:

- cheque books with bank statements
- cash register tallies with daily receipts.

[CN, PS, T]

Illustrative Examples

2.1 The following petty cash transactions occurred during the first week of March.

March 4	$100 cheque was received to establish the fund.
March 5	Bought $12.50 worth of postage stamps.
March 5	Spent $10 to have something delivered by taxi.
March 6	Spent $6.50 for lunch.
March 7	Paid a courier service $25 for deliveries.
March 7	Bought flowers for opening day, $28.
March 8	Replenished the fund by $25.
March 9	Postage stamps purchased for $21.50.

Determine if a final balance of $20 is correct. If not, provide an explanation for the difference, and indicate possible ways to correct the problem.

2.2 Complete the table below to determine the cost of credit for using a department store charge account for the period shown. Monthly credit charges are 1.4% of the balance due.

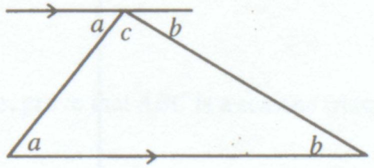
Month	Previous Balance	– Payment Made	+ Purchases Charged	⇒ Balance Due	+ Credit Charges	⇒ New Balance
February	$314.65	$100.00	$193.75		$5.72	$414.12
March		$150.00	$ 59.60			
April		$140.00	$421.83			$618.62
May	$618.62	$200.00	$ 39.65			
June		$250.00	$ 58.11			
July		$150.00	$ 77.21			
August	$206.68	$120.00	$163.09		$3.50	$253.27

Strand: Patterns and Relations (Patterns)

Students will:

- use patterns to describe the world and to solve problems.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[E]	Estimation and Mental Mathematics	**[T]**	Technology
		[V]	Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
Apply the principles of mathematical reasoning to solve problems and to justify solutions.	P5–1. (PR5) Differentiate between inductive and deductive reasoning. [CN, R]	1.1 Find, inductively, the sum of the angles of a triangle, by: a) constructing triangles and tearing the corners off b) putting the torn corners together to form a straight line. 1.2 Show, deductively, that the sum of the measures a, b and c is 180°, by: a) drawing a triangle b) using one side as a base and drawing a parallel line segment on the opposite vertex c) knowing that $a = a$, $b = b$, and c is included in both; $\therefore\ a + c + b = 180°$. 
	P5–2. (PR6) Explain and apply connecting words, such as "and", "or" and "not", to solve problems. [C, PS, R, V]	2.1 Each member of a sports club plays at least one of the following sports: soccer, rugby or tennis. The following information is given: a) 163 play tennis; 36 play tennis and rugby; 13 play tennis and soccer b) 6 play all three sports; 11 play soccer and rugby; 208 play rugby or tennis c) 98 play soccer or rugby. Use this information to determine the number of members in the club. 2.2 On a number line, indicate the location of the sets corresponding to the following: a) $x < 2$ or $x > 5$ b) $x < 2$ and $x > 5$ c) $x < 5$ or $x > 2$ d) $x < 5$ and not $x > 2$. 2.3 The phrase "*A* or *B*" can be used in ordinary speech in inclusive and exclusive senses, depending on whether "*A* and *B*" is included or excluded. a) Give a practical example of each sense of "*A* or *B*". b) Show the relationship between the inclusive and the exclusive sense of "*A* or *B*" on appropriate Venn diagrams. c) Mathematicians and logicians use the inclusive sense of "*A* or *B*". Justify this choice.
(continued)		

Strand: Number (Number Operations)

Students will:

- demonstrate an understanding of and proficiency with calculations
- decide which arithmetic operation or operations can be used to solve a problem and then solve the problem.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

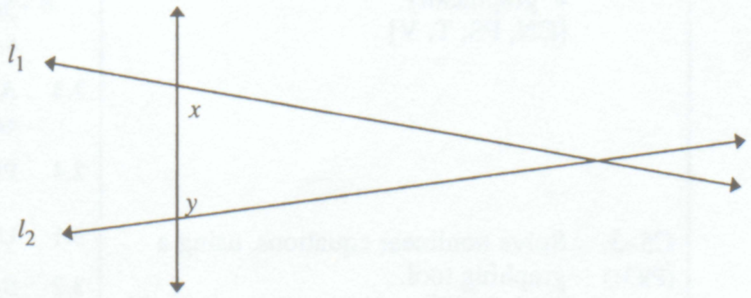
<table>
<tr><th>General Outcomes</th><th>Specific Outcomes</th><th>Illustrative Examples</th></tr>
<tr><td><i>(continued)</i></td><td><i>(continued)</i></td><td>4.2 Plot the world population on the vertical axis and the date on the horizontal axis. Use the graph to predict the date when the population reached 4 billion and to predict the present population of the world.
<table>
<tr><th>Date</th><th>Population</th></tr>
<tr><td>1650</td><td>500 000 000</td></tr>
<tr><td>1850</td><td>1 100 000 000</td></tr>
<tr><td>1930</td><td>2 000 000 000</td></tr>
<tr><td>1950</td><td>2 500 000 000</td></tr>
<tr><td>1970</td><td>3 600 000 000</td></tr>
<tr><td>1988</td><td>5 100 000 000</td></tr>
</table></td></tr>
<tr><td></td><td>C4–5. (N16) Solve investment and credit problems involving simple and compound interest.
[CN, PS, T]</td><td>5.1 Determine the effective annual interest rate on a loan of $1000 at 10% per year, compounded quarterly.
5.2 Calculate the compound amount, after one year, of a deposit of $1000. Assume the current nominal annual interest when the interest is compounded:
a) annually
b) monthly
c) daily.
5.3 A bank offers an interest rate of 8% per year, compounded annually. A second bank offers an interest rate of 8% per year, compounded quarterly. If $2000 were deposited, for ten years, in each bank, how much more income would be gained in the second bank than in the first?
5.4 Calculate the interest paid on various forms of credit, including:
a) credit cards
b) loans
c) mortgages.
5.5 A loan of $5000 carries an interest rate of 9% per year, compounded monthly. Adele makes a payment of $350 every month. Use a spreadsheet to determine how much she still owes after making 12 payments.
5.6 Compare two investments in an RRSP for one year with contributions starting January 1.
a) $100 is invested monthly at 10% per annum, compounded monthly.
b) $600 is invested semi-annually at 10% per annum, compounded semi-annually.</td></tr>
</table>

Strand: Patterns and Relations (Patterns)

Students will:

- use patterns to describe the world and to solve problems.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
(continued)	P5–4. (PR8) Distinguish between an "if–then" proposition, its converse and its contrapositive. [CN, R]	4.1 Change the statement "Multiples of 3 are always multiples of 6" into "if–then" form, and write the converse and contrapositive of the "if–then" statement. Decide on the truth of all three propositions. 4.2 Create a true proposition whose converse and contrapositive are both true.
	P5–5. (PR9) Prove assertions in a variety of settings, using direct and indirect reasoning. [R]	5.1 Angle *ABC* is obtuse, and *AD* is the median of *BC*. If *AD* is not an altitude, prove that *ABC* is a scalene triangle. 5.2 Prove that the medians of a triangle cannot bisect each other. 5.3 In the diagram below, show: a) $x + y < 180°$ b) if $x + y = 180°$, lines l_1 and l_2 are parallel.  5.4 Prove that the difference of squares of two odd numbers is always divisible by 4.

Strand: Patterns and Relations (Patterns)

Students will:

- use patterns to describe the world and to solve problems.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[E]	Estimation and Mental Mathematics	**[T]**	Technology
		[V]	Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
(continued)	P5–3. (PR7) Use examples and counterexamples to analyze conjectures. [CN, R]	3.1 Rajiv concluded that whenever he added two prime numbers the sum was always even. Find a counterexample to prove that Rajiv's conjecture is false. 3.2 A science text states that water boils at 100°C. Find a counterexample. 3.3 Mary used her graphing calculator to graph $y = x^x$. She found the screen to be blank for $x < 0$ and made a conjecture that x^x is undefined when $x < 0$. Find an example to show that Mary's conjecture is reasonable. Find a counterexample to show that Mary's conjecture is false. 3.4 The functions $f(x) = \frac{x^2 - 49}{x - 7}$ and $g(x) = x + 7$ are closely related. a) Explain the similarities and the differences between $f(x)$ and $g(x)$. b) How do the graphs of $f(x)$ and $g(x)$ differ from one another?

Strand: Patterns and Relations (Variables and Equations)

Students will:

- represent algebraic expressions in multiple ways.

[C] Communication
[CN] Connections
[E] Estimation and Mental Mathematics
[PS] Problem Solving
[R] Reasoning
[T] Technology
[V] Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
Represent and analyze situations that involve expressions, equations and inequalities.	P3–1. (PR32) Solve nonlinear equations: • by factoring • graphically. [CN, T, V]	1.1 Solve by factoring: a) $x^2 - 2x = 24$ b) $x^3 = 1$ c) $2x^2 + 9x - 5 = 0$ d) $7x^2 + 4x - 11 = 0$. 1.2 Solve each of the above graphically. For example, $x^2 - 2x = 24$ can be solved by graphing $y = x^2 - 2x$ and $y = 24$ and using the points of intersection to determine the solution. 1.3 Solve $3x^2 + 1 = 10x - 2$ graphically in two different ways. Is there one way that gives more reliable results? Explain your procedures and the results obtained.
(continued)	P3–2. (PR33) Use the Remainder Theorem to evaluate polynomial expressions and the Factor Theorem to determine factors of polynomials. [E, PS, T]	2.1 The polynomial $P(x) = 4x^3 + bx^2 + cx + 11$ has a remainder of -7 when divided by $(x + 2)$ and a remainder of 14 when divided by $(x - 1)$. Find the values of b and c. 2.2 Factor $x^3 - 2x^2 - 5x + 6$.

Strand: Patterns and Relations (Variables and Equations)

Students will:

- represent algebraic expressions in multiple ways.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[E]	Estimation and Mental Mathematics	**[T]**	Technology
		[V]	Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
Represent and analyze situations that involve expressions, equations and inequalities.	C5–1. (PR29) Graph linear inequalities, in two variables. [PS, V]	1.1 Solve, algebraically and graphically, for x: $2x + 5 > 3x - 1$. 1.2 A target is described in terms of coordinates (x, y), where x and y are measured in metres. All of the following are true: • $x \leq 6$ • $y \geq 7$ • (x, y) is in the first quadrant • $x + y \leq 10$. What is the shape and the area of the target?
	C5–2. (PR30) Solve systems of linear equations, in two variables: • algebraically (elimination and substitution) • graphically. [CN, PS, T, V]	2.1 Solve this system of equations, using the elimination method: $x + 2y = 10$ $2x + 3y = 14$. 2.2 Solve this system of equations, using the substitution method: $3x + 4y = 15$ $x - y = 5$. 2.3 A principal of $42 000 is invested partly at 7% and partly at 9.5%. If the interest is $3700, how much is invested at each interest rate? 2.4 Plot the graphs of $2x + 3y = 11$ and $2x - 3y = 17$. What is their point of intersection?
	C5–3. (PR31) Solve nonlinear equations, using a graphing tool. [CN, T, V]	3.1 Using a graphing tool, solve $x^2 + 6x - 11 = 0$. 3.2 Solve $x^3 + x = 30$ graphically, using two different methods. Which method gives solutions that are freer from rounding errors and other inaccuracies? 3.3 Where does the line $y = 4x + 5$ cut the curve $y = 2^x$? Use a graphing tool to find the points of intersection.

Strand: Patterns and Relations (Variables and Equations)

Students will:

- represent algebraic expressions in multiple ways.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[E]	Estimation and Mental Mathematics	**[T]**	Technology
		[V]	Visualization

General Outcomes	Specific Outcomes		Illustrative Examples
(continued)	P3–4. (PR35)	Solve systems of linear equations, in three variables: • algebraically • with technology. [CN, PS, T, V]	4.1 Determine the solution to the following system: $2x + y - z = 3$ $x + 2y + z = 0$ $3x - y - 2z = 11.$ 4.2 The total revenue R is a quadratic function of the price p of books sold. So $R = ap^2 + bp + c$. Find the values of a, b and c, if the revenue is \$6000 at a price of \$30, \$6000 at a price of \$40 and \$5000 at a price of \$50.

Strand: Patterns and Relations (Variables and Equations)

Students will:

- represent algebraic expressions in multiple ways.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
(continued)	P3–3. (PR34) Determine the solution to a system of nonlinear equations, using technology as appropriate. [PS, T, V]	3.1 Find the solutions to the following system: $y = x^2$ $y = 8 - x^2$. 3.2 Graphically, find the solution set to the following system: $y = 3x + 2$ and $y = 2^x$. How do you know that the solution set is complete? 3.3 The world's population grows by 2% per year. The world food production can sustain an additional 200 million people per year. In 1987 the population was 5 billion, and food production could sustain 6 billion people. The population growth can be modelled by the equation $P_1 = 5(1.02)^n$, with the food production being modelled by $P_2 = 0.2n + 6$. The variable n is the number of years after 1987. a) When does $P_1 = P_2$? b) If $P_1 > P_2$ is true, when does this happen, and how is this inequality interpreted?

Strand: Patterns and Relations (Relations and Functions)

Students will:

- use algebraic and graphical models to generalize patterns, make predictions and solve problems.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[E]	Estimation and Mental Mathematics	**[T]**	Technology
		[V]	Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
Examine the nature of relations with an emphasis on functions. *(continued)*	P4–1. (PR54) Perform operations on functions and compositions of functions. [CN, E, PS]	1.1 If $f(x) = 3x + 2$ and $g(x) = x^2$, find: a) $3f(x)$ b) $f(x) \bullet g(x)$ c) $f(x) + g(x)$ d) $f(g(x))$ e) $f(f(x))$. 1.2 A ball thrown in the air has a velocity given by $v(t) = 49 - 9.8t$. The kinetic energy function $K(v)$ is given by $K(v) = 0.4v^2$. Express the ball's kinetic energy as a function $K(t)$ of time.

Strand: Patterns and Relations (Relations and Functions)

Students will:

- use algebraic and graphical models to generalize patterns, make predictions and solve problems.

[C] Communication
[CN] Connections
[E] Estimation and Mental Mathematics
[PS] Problem Solving
[R] Reasoning
[T] Technology
[V] Visualization

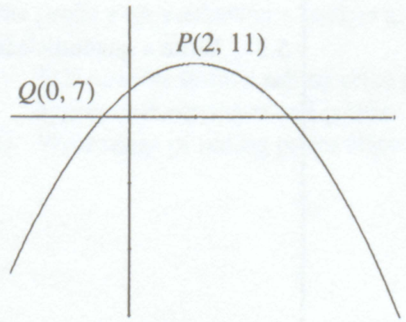
General Outcomes	Specific Outcomes	Illustrative Examples
Represent and analyze quadratic, polynomial and rational functions, using technology as appropriate.	C5–4. (PR57) Determine the following characteristics of the graph of a quadratic function: • vertex • domain and range • axis of symmetry • intercepts. [C, PS, T, V]	4.1 Given the graph of any quadratic function, determine the following: a) vertex b) domain c) range d) axis of symmetry e) intercepts. 4.2 Use technology to graph $f(x) = x^2 - 6x + 4$ and to determine the vertex, domain, range, axis of symmetry and intercepts. 4.3 One model concerning the rate of population growth of Earth has the annual rate of increase varying jointly as the population and the unused carrying capacity of Earth. The equation of the model is: $y = 0.001x(21 - x)$, where y = the rate of increase in population (in billions per year), and x = the present population (in billions). a) Plot this model of growth. b) The present population of Earth is 5.8 billion. What is the annual increase in population at present? c) What is the population when the rate of increase in population is at its greatest? d) What is the population when the rate of increase is zero? e) What is the projected maximum population that Earth can accommodate, according to this model?

Strand: Patterns and Relations (Relations and Functions)

Students will:

- use algebraic and graphical models to generalize patterns, make predictions and solve problems.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[E]	Estimation and Mental Mathematics	**[T]**	Technology
		[V]	Visualization

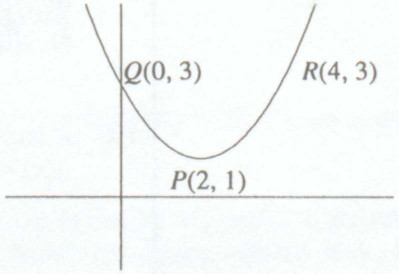
General Outcomes	Specific Outcomes		Illustrative Examples
Represent and analyze quadratic, polynomial and rational functions, using technology as appropriate.	P4–3. (PR58)	Connect algebraic and graphical transformations of quadratic functions, using completing the square as required. [CN, T, V]	3.1 Graph $f(x) = 2x^2 + 5x - 7$. 3.2 Give a list of events or situations that might be described by a quadratic, parabolic, shape. 3.3 Given the graph of $y = x^2$, sketch $y = -2(x - 3)^2 - 4$. 3.4 Given the graph of $y = x^2$, what is the equation for the transformed graph shown here?  3.5 Rewrite the equation of $f(x) = 2x^2 - 12x + 13$ in the form $f(x) = a(x - p)^2 + q$, and graph the function.
(continued)	P4–4. (PR59)	Model real-world situations, using quadratic functions. [CN, PS]	4.1 Computer software programs are sold to students for \$20 each, and 300 students are willing to buy them at that price. For every \$5 increase in price, there are 30 fewer students willing to buy the software. What is the maximum revenue? 4.2 What is the maximum rectangular area that can be enclosed by 120 m of fencing, if one of the sides of the rectangle is an existing wall?

Strand: Patterns and Relations (Relations and Functions)

Students will:

- use algebraic and graphical models to generalize patterns, make predictions and solve problems.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[E]	Estimation and Mental Mathematics	**[T]**	Technology
		[V]	Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
(continued)	P4–2. Determine the inverse of a function. (PR55) [CN, R, V]	2.1 Graph the inverse of $y = \frac{x}{(x-1)}$, and determine the equation, domain and range of the inverse. 2.2 Sketch the inverse of the following.  2.3 Sketch the inverse of $f(x) = x^2$. 2.4 If $f(x) = 2x - 1$ and $g(x) = \frac{x+1}{2}$, find $f(g(x))$ and $g(f(x))$, and show that the functions $f(x)$ and $g(x)$ are inverses of each other. 2.5 Determine the domain and range for each of the functions in illustrative examples 9.2 and 9.3. 2.6 Is the inverse of $f(x) = 2x - 5$ a function?

Strand: Patterns and Relations (Relations and Functions)

Students will:

- use algebraic and graphical models to generalize patterns, make predictions and solve problems.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[E]	Estimation and Mental Mathematics	**[T]**	Technology
		[V]	Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
(continued)	P4–6. (PR61) Determine the character of the real and non-real roots of a quadratic equation, using: • the discriminant in the quadratic formula • graphing. [C, R, T, V]	6.1 If $3x^2 - mx + 2 = 0$ can be factored, what values of m are possible? 6.2 Discuss the implications of a negative discriminant when describing the zeros of a quadratic function. 6.3 Given $3x^2 - mx + 3 = 0$: a) For what value(s) of m would one root be double the other? b) For what values of m would the roots not be real? 6.4 The profit y for publishing a book is given by the equation $y = -5x^2 + 400x - 3000$, where x is the selling price per book. a) Is it possible to set a selling price that will earn a total profit of $6000? Explain your solution with reference to appropriate equations and graphs. b) What range of selling prices allow the publisher to make a profit on this book?

Strand: Patterns and Relations (Relations and Functions)

Students will:

- use algebraic and graphical models to generalize patterns, make predictions and solve problems.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[E]	Estimation and Mental Mathematics	**[T]**	Technology
		[V]	Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
(continued)	P4–5. (PR60) Solve quadratic equations, and relate the solutions to the zeros of a corresponding quadratic function, using: • factoring • the quadratic formula • graphing. [CN, E, T, V]	5.1 Solve $3x^2 - 5x + 2 = 0$ algebraically and by graphing the corresponding function $f(x) = 3x^2 - 5x + 2$. 5.2 When bicycles are sold for $280 each, a cycle store can sell 80 in a season. For every $10 increase in the price, the number sold drops by 3. a) Represent the sales revenue as a quadratic function of either the number sold or the price. b) What is the number sold, and the price, if the total sales revenue is exactly $20 000? c) What is the range of prices that will give a sales revenue that exceeds $15 000? 5.3 Write a quadratic equation whose roots are $\frac{3}{2}$ and $-\frac{1}{4}$. Is this equation unique?

Strand: Patterns and Relations (Relations and Functions)

Students will:

- use algebraic and graphical models to generalize patterns, make predictions and solve problems.

[C] Communication
[CN] Connections
[E] Estimation and Mental Mathematics
[PS] Problem Solving
[R] Reasoning
[T] Technology
[V] Visualization

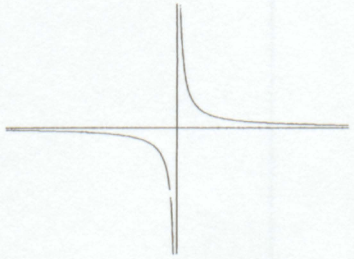
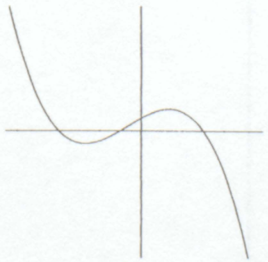
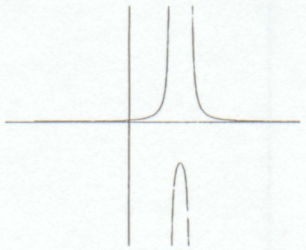
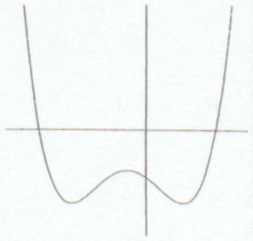
General Outcomes	Specific Outcomes	Illustrative Examples
(continued)	*(continued)*	7.3 Graph $y = x^2(x^2 - 4)$. What is the domain and range of this function?
		7.4 Graph $y = x^2 - 1$, identify the zeros of this function, and use these to predict the asymptotes of $y = \frac{1}{(x^2-1)}$. Then graph $y = \frac{1}{(x^2-1)}$, using a graphing tool. Compare the two graphs, considering domain, range, asymptotes and zeros.
		7.5 Use a graphing tool to graph $y = \frac{x^2}{(x^2-4)}$ and to predict the domain, range and zeros. Describe the symmetry.
		7.6 Use technology to graph $f(x) = x^3 - 4x^2 + k$ for various values of k. a) Estimate the values of k for which the equation $f(x) = 0$ appears to have a double root. b) Show that $k = 0$ ensures that $f(x) = 0$ has a double root. c) Show that $k = \frac{256}{27}$ ensures that $f(x) = 0$ has a double root.
	P4–8. (PR63) Formulate and apply strategies to solve absolute value equations, radical equations, rational equations and inequalities. [CN, R, V]	8.1 Sketch $f(x) = \|x-1\| - 4$, and determine the values of x for which $f(x) > 0$.
		8.2 Solve for x: a) $\|x-1\| > 7$ b) $\sqrt{(x-1)} + \sqrt{(x+4)} = 5$ c) $\sqrt{(x+2)} > \frac{x}{x+2}$ d) $\|x-1\| + \|2x-1\| > 7$.
		8.3 The point P lies on the y-axis, while points A and B are (–9, 0) and (5, 0) respectively. If $PA + PB$ is 28 units long, determine the coordinates of P.

Strand: Patterns and Relations (Relations and Functions)

Students will:

- use algebraic and graphical models to generalize patterns, make predictions and solve problems.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[E]	Estimation and Mental Mathematics	**[T]**	Technology
		[V]	Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
(continued)	P4–7. (PR62) Describe, graph and analyze polynomial and rational functions, using technology. [C, R, T, V] *(continued)*	7.1 Determine if each of the following examples is a rational function, a polynomial function or some other type of function, and justify your conclusion. a) $y = x^2 - 3x + \sqrt{7}$ b) $y = (x-5)^{-1}$ c) $y = \frac{1}{5}x^4 + 3x^3 - 12x - 0.75$ d) $y = \sqrt{7x^5} + x^2$ e) $y = 2^x - 9$ f) $y = \frac{3x-7}{x^2 - 5x + 6}$ 7.2 Examine the following graphs. Which could be graphs of rational functions, and which could be graphs of polynomial functions? a)  b)  c)  d) 

Strand: Shape and Space (3-D Objects and 2-D Shapes)

Students will:

- describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

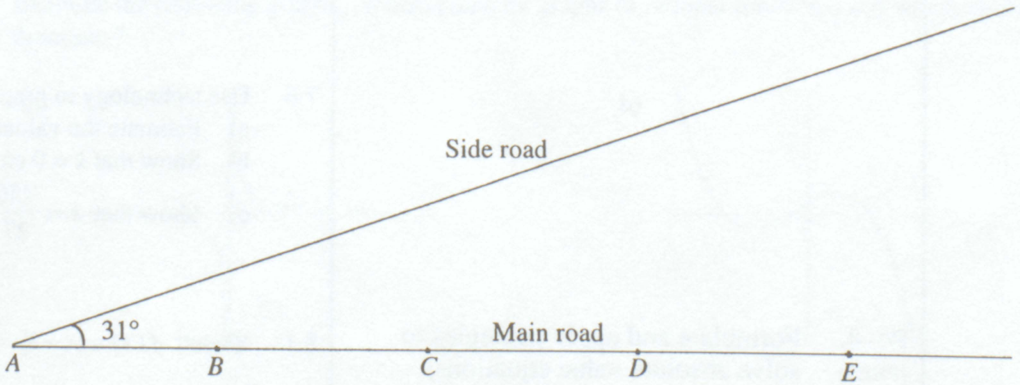
General Outcomes	Specific Outcomes	Illustrative Examples
Solve coordinate geometry problems involving lines and line segments, and justify the solutions.	P3–6. (SS24) Solve problems involving distances between points and lines. [CN, PS, R]	6.1 Determine the shortest distance from (3, 4) to the line $2x - 5y = 7$. 6.2 The lines $y = 3x + 1$ and $y = 3x - 9$ are parallel. Determine the vertical distance between the two lines, the horizontal distance between the two lines and the shortest distance between the two lines.
	P3–7. (SS25) Verify and prove assertions in plane geometry, using coordinate geometry. [C, R, V]	7.1 Given $A = (-1, 3)$, $B = (0, 5)$ and $C = (-2, 6)$: a) Verify that ABC is a right-angled triangle. b) Is ABC isosceles? Justify your assertion. c) If M is the midpoint of AB and N is the midpoint of AC, prove that MN is parallel to BC. d) Find a point D so that $ABCD$ is a parallelogram. Prove that $ABCD$ is not a rectangle. 7.2 Use coordinate geometry to prove that: a) the diagonals of any parallelogram bisect one another b) if ABC is any triangle, with M as the midpoint of AB and N as the midpoint of AC, then MN is parallel to BC and is half its length. 7.3 Use coordinate geometry to divide the line segment with end points $A(4, 7)$ and $B(-3, 8)$ into five congruent parts.

Strand: Shape and Space (Measurement)

Students will:

- describe and compare everyday phenomena, using either direct or indirect measurement.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

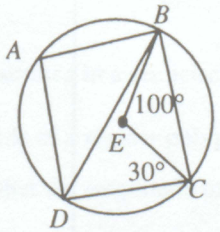
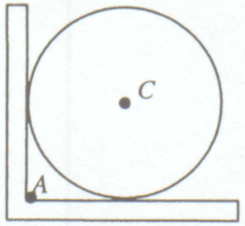
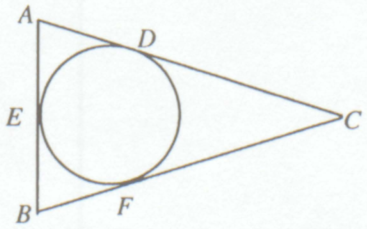
General Outcomes	Specific Outcomes	Illustrative Examples
Solve problems involving triangles, including those found in 3-D and 2-D applications.	P3–5. (SS7) Solve problems involving ambiguous case triangles in 3-D and 2-D. [CN, PS, R, T]	5.1 An 11 cm long line AB is drawn at an angle of 44° to a horizontal line AE. A circle with centre B and a radius of 9 cm is drawn, cutting the horizontal line at points C and D. Calculate the length of the chord CD. 5.2 The line segment of equation $y = 2.4x$, passes through $A(0, 0)$ and $C(5, 12)$, has a length of 13 and makes an angle of 67.3° with the horizontal x-axis. a) What points are located with $CB = 10$ and AB horizontal? b) Check your answer by determining the intersection points of the circle $(x-5)^2 + (y-12)^2 = 100$ and the line $y = 0$. c) Use a suitable diagram to explain why the answers to a) and b) are the same. 5.3  Streetlights A, B, C, D and E are placed 50 m apart on the main road, as indicated on the diagram. The light from a streetlight can travel 24 m. Determine the furthest point on the side road that is lighted and the length of side road that is illuminated by both streetlight C and streetlight D.

Strand: Shape and Space (3-D Objects and 2-D Shapes)

Students will:

- describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[E]	Estimation and Mental Mathematics	**[T]**	Technology
		[V]	Visualization

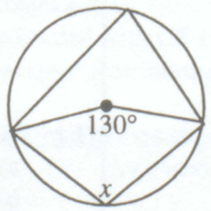
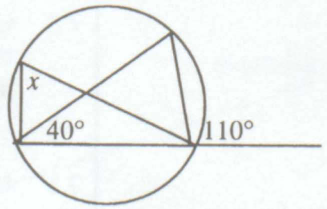
General Outcomes	Specific Outcomes	Illustrative Examples
(continued)	*(continued)*	5.5 Determine the measure of $\angle ECB$, $\angle BDC$, $\angle BAD$ and $\angle DBE$, where E is the centre of the circle.  5.6 How far from the inside corner of the shelf, A, is the centre C of the plate, if the plate has a diameter of 20 cm?  5.7 The perimeter of the isosceles triangle ABC, with $AC = BC$, is 54 cm. If $AD = 5$ cm, and D, E and F are points of tangency, find the length of BC. 

Strand: Shape and Space (3-D Objects and 2-D Shapes)

Students will:

- describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[E]	Estimation and Mental Mathematics	**[T]**	Technology
		[V]	Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
Develop and apply the geometric properties of circles and polygons to solve problems. *(continued)*	C5–5. (SS26) Use technology and measurement to confirm and apply the following properties to particular cases: • the perpendicular from the centre of a circle to a chord bisects the chord • the measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc • the inscribed angles subtended by the same arc are congruent • the angle inscribed in a semicircle is a right angle • the opposite angles of a cyclic quadrilateral are supplementary • a tangent to a circle is perpendicular to the radius at the point of tangency • the tangent segments to a circle, from any external point, are congruent • the angle between a tangent and a chord is equal to the inscribed angle on the opposite side of the chord • the sum of the interior angles of an n-sided polygon is $(2n - 4)$ right angles. [PS, R, T, V] *(continued)*	5.1 A plate, with a diameter of 20 cm, is placed on a square place mat, with no overhang. Calculate the length of the diagonal of the square. 5.2 Determine the measure of angle x.  5.3 Determine the measure of angle x.  5.4 Draw a semicircle with diameter AB. Draw an angle, ACB, with C being any point on the semicircle. What is the measure of angle ACB? Repeat for two other points, C' and C'', on the semicircle. What pattern emerges?

Strand: Shape and Space (3-D Objects and 2-D Shapes)

Students will:

- describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[E]	Estimation and Mental Mathematics	**[T]**	Technology
		[V]	Visualization

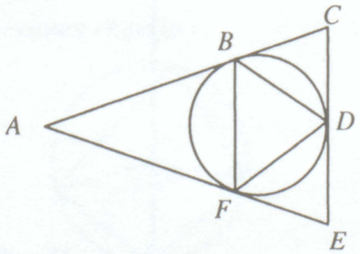
General Outcomes	Specific Outcomes	Illustrative Examples
Develop and apply the geometric properties of circles and polygons to solve problems. *(continued)*	P5–6. (SS28) Prove the following general properties, using established concepts and theorems: • the perpendicular bisector of a chord contains the centre of the circle • the measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc (for the case when the centre of the circle is in the interior of the inscribed angle) • the inscribed angles subtended by the same arc are congruent • the angle inscribed in a semicircle is a right angle • the opposite angles of a cyclic quadrilateral are supplementary • a tangent to a circle is perpendicular to the radius at the point of tangency • the tangent segments to a circle from any external point are congruent • the angle between a tangent and a chord is equal to the inscribed angle on the opposite side of the chord • the sum of the interior angles of an n-sided polygon is $(2n-4)$ right angles. [C, R, V]	6.1 a) For what values of c does the line $y = c$ touch the circle $x^2 + y^2 = r^2$? b) Use the result from part a) to show that the tangent to a circle is perpendicular to the radius at the point of tangency. 6.2 Show that the angle inscribed in a semicircle is a right angle. 6.3 The chord AB is one side of a regular polygon of n sides. The polygon is inscribed in a circle. If D is any other vertex of the polygon, prove that the magnitude of angle ADB is $\frac{180°}{n}$.

Strand: Shape and Space (3-D Objects and 2-D Shapes)

Students will:

- describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[E]	Estimation and Mental Mathematics	**[T]**	Technology
		[V]	Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
(continued)	*(continued)*	5.8 Determine the measure of $\angle CAE$, if $\angle BDF = 60°$ and $\angle FDE = 70°$. 

MATHEMATICS 12

derived from

The Common Curriculum Framework

for

K–12 MATHEMATICS

Grade 10 to Grade 12

Western Canadian Protocol for Collaboration in Basic Education

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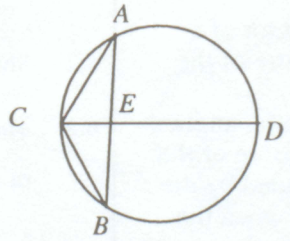
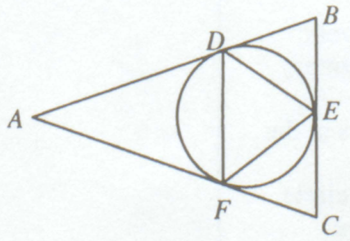
JUNE 1996

Strand: Shape and Space (3-D Objects and 2-D Shapes)

Students will:

- describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[E]	Estimation and Mental Mathematics	**[T]**	Technology
		[V]	Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
(continued)	P5–7. (SS29) Solve problems, using a variety of circle properties, and justify the solution strategy used. [PS, R, V]	7.1 If diameter CD is perpendicular to chord AB at E, prove that triangle ABC is isosceles.  7.2 Determine the measure of $\angle BAC$, if $\angle DEF = 60°$ and $\angle EFC = 70°$. Provide a reason for each step in the solution strategy.  7.3 A chain on a bicycle connects two gear wheels of diameters 9 cm and 19 cm respectively. The centres of the gear wheels are 87 cm apart. Find the minimum length of the chain.

MATHEMATICS 12: GENERAL OUTCOMES, AND SPECIFIC OUTCOMES WITH ILLUSTRATIVE EXAMPLES, ORGANIZED BY STRAND AND SUBSTRAND

This section elaborates on the general outcomes and specific outcomes by providing illustrative examples, by strand and substrand, for the Mathematics 12 course.

The coding for mathematical processes follows the same scheme as in the *Common Curriculum Framework.*

CLUSTERS IN THE MATHEMATICS 12 COURSE

There are 5 clusters identified, each representing 20 to 25 hours of instructional time for an average student taking the cluster.

The common cluster, numbered C6, is part of the mathematics expected of all students completing a K to 12 mathematics program.

Pure clusters, numbered P6 to P9, place more emphasis on precise mathematical theory. The approaches used are primarily algebraic and graphical.

CODING FOR ILLUSTRATIVE EXAMPLES (IEs)

The illustrative examples (IEs) listed on the following pages are organized by strand and substrand and have been correlated to specific outcomes (SOs). The numbers are taken directly from the *Common Curriculum Framework.*

NUMBERING SYSTEM

The specific outcomes are cross-referenced to the General Outcomes and Specific Outcomes section (pages 30 to 59 of the *Common Curriculum Framework*). For example, $\underset{\text{(PR53)}}{\text{C2 – 6.}}$ is the 6th specific outcome in Common Cluster 2 and the 53rd specific outcome in the Patterns and Relations strand.

Strand: Patterns and Relations (Patterns)

Students will:

- use patterns to describe the world and to solve problems.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[E]	Estimation and Mental Mathematics	**[T]**	Technology
		[V]	Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
Generate and analyze exponential patterns. *(continued)*	P6–1. (PR19) Derive and apply expressions to represent general terms and sums for geometric growth and to solve problems. [CN, R, T]	1.1 Determine the n^{th} term and the sum of the first n terms of the geometric sequence whose first three terms are 2, 6 and 18. 1.2 Mathematicians use sigma notation as a way to write the sum of a series. For example: $\sum_{k=1}^{5} 2^k = 2^1 + 2^2 + 2^3 + 2^4 + 2^5$. Use sigma notation to write the series 5 – 15 + 45 – . . . + 3645. 1.3 Suppose that a principal of P dollars is invested at an annual interest rate r that is compounded annually. The amount A after t years is given by $A = P(1 + r)^t$. a) Find the number of years for the amount to double, if $2000 is invested at a rate of 7.5%, compounded annually. b) If the interest rate were 7.25% per annum, compounded semi-annually, how would the doubling period change? c) What would be the doubling period, if the rate were 7% per annum, compounded daily? 1.4 For the geometric series $1-\frac{1}{3}+\frac{1}{9}-\frac{1}{27}+\ldots$, find the sum of 20 terms. 1.5 The time needed for an investment to double in value can be estimated using the rule of 72, which states that $n = \frac{72}{i}$ where i is the annual percentage interest rate and n the number of years. a) Compare the rule of 72 doubling time with the exact doubling time for the following interest rates: • 4% per annum, compounded annually • 8% per annum, compounded annually • 24% per annum, compounded annually. b) What general conclusion can be drawn as to the accuracy of rule of 72 calculations?

Strand: Patterns and Relations (Variables and Equations)

Students will:

- represent algebraic expressions in multiple ways.

[C] Communication
[CN] Connections
[E] Estimation and Mental Mathematics
[PS] Problem Solving
[R] Reasoning
[T] Technology
[V] Visualization

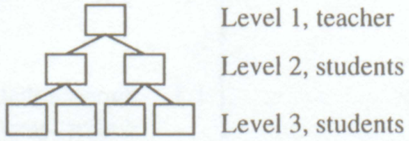
General Outcomes	Specific Outcomes	Illustrative Examples
Solve exponential, logarithmic and trigonometric equations and identities.	P6–4. (PR39) Solve exponential equations having bases that are powers of one another. [E, R]	4.1 Solve for *x*: $3^{(4x-1)} = 27^{2x}$.
		4.2 A string of ones and zeros is the binary representation of a number. If this number is converted to the base-16 hexadecimal representation, it is 9 digits shorter. As well, the decimal and hexadecimal representations have the same number of digits. a) How many digits are there in the binary representation of the original number? b) Between what two decimal numbers does the original number lie?
	P6–5. (PR40) Solve and verify exponential and logarithmic equations and identities. [R]	5.1 Solve for *x*: $\log_2(x-2) + \log_2(x) = \log_2(3)$.
		5.2 Solve for *x*: $2 \times 3^x = 5^{(x-1)}$.
		5.3 Solve for *x*, checking for any extraneous solutions: $\log_5(3x+1) + \log_5(x-3) = 3$.
		5.4 The pH of an acid is given by $\text{pH} = -\log_{10}[H^+]$, where $[H^+]$ is the hydrogen ion concentration in moles per litre. What is the hydrogen ion concentration of a weak vinegar solution of pH = 3.1?
		5.5 Joe has $50 000 invested at an interest rate of 7% per annum, compounded monthly. Laura has $40 000 invested at 9.5% per annum, compounded annually. After how many years will the two investments be equal in value?
		5.6 Verify the identity $\log_a\left(\frac{1}{x}\right) = -\log_a x$ for any base *a* and any positive value of *x*.

Strand: Patterns and Relations (Patterns)

Students will:

- use patterns to describe the world and to solve problems.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

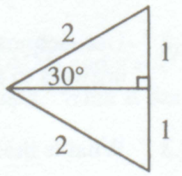
General Outcomes	Specific Outcomes	Illustrative Examples
(continued)	P6–2. (PR20) Connect geometric sequences to exponential functions over the natural numbers. [E, R, V]	2.1 The world's population grows by 2% per year. The world food production can sustain an additional 200 million people per year. In 1987 the population was 5 billion, and food production could sustain 6 billion people. a) Calculate the population in 1998, 2009, 2019. b) Calculate the population that food production could sustain in 1998, 2009, 2019. c) When will the population exceed the food supply? 2.2 The following is a school trip telephoning tree.  a) At what level are 64 students contacted? b) How many are contacted at the 8th level? c) By the 8th level how many students, in total, have been contacted? d) By the *n*th level how many students, in total, have been contacted? e) If there are 300 students in total, by what level will all have been contacted?
	P6–3. (PR21) Estimate values of expressions for infinite geometric processes. [PS, R, T]	3.1 For the infinite series $2+\frac{2}{5}+\frac{2}{25}+\ldots$, estimate the sum to four decimal places. 3.2 An oil well produces 25 000 barrels of oil during its first month of production. If its production drops by 5% each month, estimate the total production before the well runs dry.

Strand: Patterns and Relations (Variables and Equations)

Students will:

- represent algebraic expressions in multiple ways.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
(continued)	P8–2. (PR42) Determine the exact and the approximate values of trigonometric ratios for any multiples of 0°, 30°, 45°, 60° and 90° and 0, $\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$. [CN, E]	2.1 Given an equilateral triangle with a side of 2 units, determine the exact trigonometric ratios of 30°.  2.2 Find the exact values for $\sin\frac{7\pi}{6}$, $\tan\frac{2\pi}{3}$, $\cos\frac{7\pi}{4}$.
	P8–3. (PR43) Solve first and second degree trigonometric equations over a domain of length 2π: • algebraically • graphically. [PS, T]	3.1 Find, algebraically and graphically, the solution to the following trigonometric equations: a) $1 + 2\cos x = 5\cos x$; $0 \le x < 2\pi$. Give solutions in decimal form. b) $\sin^2 x - \sin x = 0$; $0 \le x < 2\pi$. Give solutions as exact values. c) $\cos 4x = 0.5$; $0 \le x < 2\pi$. Give solutions as exact values.

Strand: Patterns and Relations (Variables and Equations)

Students will:

- represent algebraic expressions in multiple ways.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[E]	Estimation and Mental Mathematics	**[T]**	Technology
		[V]	Visualization

General Outcomes

Solve exponential, logarithmic and trigonometric equations and identities.

(continued)

Specific Outcomes

P8–1. (PR41) Distinguish between degree and radian measure, and solve problems, using both. [CN, E]

Illustrative Examples

1.1 Draw an angle of one radian, and show how its radius and arc length are related.

1.2 Convert the following angles to degrees: $\frac{2\pi}{3}$, 1.6 rad.

1.3 Convert the following angles to radians expressed in terms of π: 180°, 55°.

1.4 In an experiment to verify the law of refraction, measurements were made of the angles of incidence and refraction. A spreadsheet was used to calculate the sines of both angles and the ratio of the two sines. The results of the spreadsheet calculation are shown in the following table.

Angle of incidence i (degrees)	Angle of refraction r (degrees)	sin i	sin r	(sin i)/(sin r)
10	7	–0.544	0.657	–0.83
20	13	0.913	0.420	2.17
30	19	–0.988	0.150	–6.59
40	25	0.745	–0.132	–5.63
50	30	–0.262	–0.988	0.27
60	35	–0.305	–0.428	0.71
70	38	0.774	0.296	2.61
80	40	–0.994	0.745	–1.33

a) In the calculations of sin i and sin r, are the angles taken as being in degrees or in radians?

b) Modify the spreadsheet so that all entries reflect radian measure.

c) Modify the spreadsheet so that all entries reflect degree measure.

d) What conclusion can be drawn from either b) or c)?

Strand: Patterns and Relations (Relations and Functions)

Students will:

- use algebraic and graphical models to generalize patterns, make predictions and solve problems.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
Represent and analyze exponential and logarithmic functions, using technology as appropriate.	P6–6. (PR64) Graph and analyze an exponential function, using technology. [R, T, V]	6.1 Graph $y = 2^x$ with/without technology. 6.2 Graph $y = 4(2^x)$ and $y = 2^x$ on the same set of coordinate axes. Identify the domain, range, asymptotes and intercepts of each graph. What is the relationship between the two graphs?
	P6–7. (PR65) Model, graph and apply exponential functions to solve problems. [PS, T, V]	7.1 The summertime population of gophers in a field can be modelled by the equation $P = 100(1.1)^n$, where n is measured in years. Plot the graph for a 10-summer period, and use the graph to find out how long it takes for the gopher population to double. 7.2 The half-life of sodium-24 is 14.9 hours. Suppose that a hospital buys a 40 mg sample of sodium-24. a) How much of the sample will remain after 48 hours? b) How long will it be until only 1 mg remains? 7.3 The population of a certain country is 28 million and grows at a rate of 3% annually. Assuming the population is continuously growing, the population P, in millions, t years from now can be determined by the formula $P = 28e^{0.03t}$ a) In how many years will the population be 40 million? b) What factors could contribute to the breakdown of this model?
	P6–8. (PR66) Change functions from exponential form to logarithmic form and vice versa. [CN]	8.1 Rewrite $y = 2^x$ as a logarithmic function. 8.2 The ionization of pure water is shown in the equations: $[H^+]\,[OH^-] = 1.0 \times 10^{-14}$ and $[H^+] = [OH^-]$. If the pH of any solution is defined as pH = $-\log_{10}[H^+]$, what is the pH of pure water?
(continued)	P6–9. (PR67) Use logarithms to model practical problems. [CN, PS, V]	9.1 Research the strength of earthquakes, and compare them, using the Richter scale. 9.2 The Armenian earthquake, Richter scale 6.9, produced 3.5×10^{13} J of energy. How much energy did the Alaska earthquake, Richter scale 8.2, produce?

Strand: Patterns and Relations (Variables and Equations)

Students will:

- represent algebraic expressions in multiple ways.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
(continued)	P8–4. (PR44) Determine the general solutions to trigonometric equations where the domain is the set of real numbers. [PS, T]	4.1 Sketch the graph of $y = \sin 3x$. Use the graph to find all solutions of $\sin 3x = 0$ in the interval $0 \leq x < 2\pi$. 4.2 Use technology to graph $y = x - 2 \sin x$, and use the graph to find all solutions to the equation $2 \sin x = x$. Express answers to a three-decimal place accuracy. 4.3 What is the relation between the graphs of $y = \sin x$ and $y = \frac{1}{2}$ and the roots of the equation $0 = 2 \sin x - 1$? 4.4 Use technology to solve $\sin 3x = \frac{1}{2}$, and then write the general solution.
	P8–5. (PR45) Verify trigonometric identities: • numerically for any particular case • algebraically for general cases • graphically. [PS, R, T, V]	5.1 a) Verify that $\sin^2 x + \cos^2 x = 1$ for any real number x. b) Use this identity to show that $1 + \tan^2 x = \sec^2 x$ for any real number x, where $\cos x \neq 0$. 5.2 Given the identity $\frac{\sin x}{1 - \cos x} = \frac{1 + \cos x}{\sin x}$: a) verify the identity for the particular case when $x = \frac{\pi}{3}$ b) verify for a general angle, using an algebraic approach c) verify, by graphing the left-hand side and the right-hand side of the given identity.
	P8–6. (PR46) Use sum, difference and double angle identities for sine and cosine to verify and simplify trigonometric expressions. [R, T]	6.1 Write 2 (sin 5)(cos 5) in terms of a single trigonometric function. 6.2 Graph the function $f(x) = \frac{2 \tan x}{1 + \tan^2 x}$. a) Make a conjecture for the period of the above graph. b) Simplify the expression for $f(x)$ to a single trigonometric function, and then find the period of $f(x)$. c) Compare the solutions to a) and b).

Strand: Patterns and Relations (Relations and Functions)

Students will:

- use algebraic and graphical models to generalize patterns, make predictions and solve problems.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
Represent and analyze trigonometric functions, using technology as appropriate.	P8–7. (PR70) Describe the three primary trigonometric functions as circular functions with reference to the unit circle and an angle in standard position. [PS, R, V]	7.1 Triangle OBA has vertices $O(0, 0)$, $B(4, 0)$ and $A(4, 3)$. The unit circle, centred at $(0, 0)$, intersects OA at point P. a) Use similar triangles to find the coordinates of point P. b) Use trigonometric ratios to find the sine and cosine of angle AOB. c) Compare your results in b) to the coordinates of point P.
	P8–8. (PR71) Draw (using technology), sketch and analyze the graphs of sine, cosine and tangent functions, for: • amplitude, if defined • period • domain and range • asymptotes, if any • behaviour under transformations. [CN, T, V]	8.1 Using a graphing utility, graph $y = \sin x$ and $y = \cos x$ on the same set of axes. a) What relationship seems to exist between the two? b) What is the amplitude and period of each graph? 8.2 Graph $y = \tan x$ and $y = \tan 2x$. Compare the period, the domain and the range of $y = \tan x$ to those of $y = \tan 2x$. 8.3 In the equation $y = A \sin [B(x + C)] + D$; $A = 4$, $B = 3$, $C = \frac{-3\pi}{4}$ and $D = -3$. Compare the graph of this function to the graph of $y = \sin x$ with respect to domain, range, amplitude, period, x and y intercepts, horizontal phase shift and vertical displacement.
	P8–9. (PR72) Draw (using technology) and analyze the graphs of secant, cosecant and cotangent functions, for: • period • domain and range • asymptotes • behaviour under transformations. [CN, T, V]	9.1 Graph and analyze: a) $y = \sec x$ b) $y = \csc x$ c) $y = \cot x$. 9.2 Compare the domain, range and period of: a) $f(x) = \csc x$ and $g(x) = 5 \csc x$ b) $f(x) = \cot x$ and $g(x) = \cot 2x$.
(continued)		

Strand: Patterns and Relations (Relations and Functions)

Students will:

- use algebraic and graphical models to generalize patterns, make predictions and solve problems.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
(continued)	P6–10. (PR68) Explain the relationship between the laws of logarithms and the laws of exponents. [C, T]	10.1 Explain how the exponent law $a^x \times a^y = a^{(x+y)}$ is related to the logarithmic law $\log_a(MN) = \log_a M + \log_a N$. 10.2 Use a calculator to find $\log_5 8$, and justify your procedure.
	P6–11. (PR69) Graph and analyze logarithmic functions with and without technology. [R, T, V]	11.1 Graph $y = \log_{10} x$ and $y = \log_2 x$ on the same set of coordinate axes. What is the likely position of the graph of $y = \log_5 x$? 11.2 Analyze the graph of $y = \log_{10}(2x + 3)$. Identify the domain, range, asymptotes and intercepts.

Strand: Shape and Space (3-D Objects and 2-D Shapes)

Students will:

- describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
Classify conic sections, using their shapes and equations.	P9–1. (SS35) Classify conic sections according to shape. [C, R, V]	1.1 Visualize the shapes generated from the intersection of a double-napped cone and a plane. For each conic section, describe the relationship between the plane, the central axis of the cone and the cone's generator.
	P9–2. (SS36) Classify conic sections according to a given equation in general or standard (completed square) form (vertical or horizontal axis of symmetry only). [CN, T, V]	2.1 A circle with a radius of 4 units has the equation $x^2 + y^2 - 16 = 0$. What are the values of A, C and F in the general form? What is the radius of the circle $25x^2 + 25y^2 - 100 = 0$? 2.2 a) Graph the circle $x^2 + y^2 = 25$. b) Graph $Ax^2 + y^2 = 25$ where $A > 1$. c) Graph $Ax^2 + y^2 = 25$ where $0 < A < 1$. d) Graph $Ax^2 + y^2 = 25$ where $A = 0$. e) Graph $x^2 + Cy^2 = 25$ where $C > 1$. f) Graph $x^2 + Cy^2 = 25$ where $0 < C < 1$. g) Graph $x^2 + Cy^2 = 25$ where $C = 0$. h) Draw a conclusion based on the results found in b) through g). 2.3 Graph $2x^2 + y^2 - 12 = 0$, using technology. Graph two other equations of this type, by changing one of the coefficients. What shape is represented by this type of graph? 2.4 Graph $4x^2 - 25y^2 - 100 = 0$, using technology. Graph two other equations of this type, by changing one of the coefficients. What shape is represented by this type of graph?
	P9–3. (SS37) Convert a given equation of a conic section from general to standard form and vice versa. [R, T]	3.1 Convert to standard form: a) $x^2 + y^2 + 6x - 8y = 11$ b) $3x^2 + y^2 + 6x + 4y = 9$. 3.2 Convert to general form: a) $\frac{(x-4)^2}{9} + \frac{(y+2)^2}{16} = 1$ b) $\frac{(x+3)^2}{25} - \frac{(y-4)^2}{16} = 1$.

Strand: Patterns and Relations (Relations and Functions)

Students will:

- use algebraic and graphical models to generalize patterns, make predictions and solve problems.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[E]	Estimation and Mental Mathematics	**[T]**	Technology
		[V]	Visualization

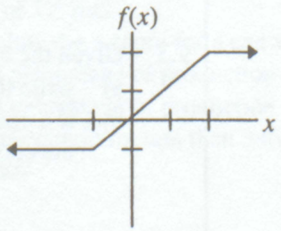
General Outcomes	Specific Outcomes	Illustrative Examples
(continued)	P8–10. Use trigonometric functions to model and solve problems. (PR73) [PS, R, V]	10.1 For a Saskatchewan town, the latest sunrise time is on December 21, at 09:15. The earliest sunrise time is on June 21, at 03:15. Sunrise times on other dates can be predicted from a sinusoidal equation. **Note**: There is no Daylight Saving Time in Saskatchewan. a) What is the equation that describes sunrise times? b) What is the amplitude and period of the equation describing sunrise times? c) Use the equation to predict the time of sunrise on April 9. d) What is the average time of sunrise throughout the year?
		10.2 The depth of water in a harbour is given by the equation $d(t) = -4.5\cos(0.16\pi t) + 13.7$, where $d(t)$ is the depth, in metres, and t is the time, in hours, after low tide. a) Sketch the graph of $d(t)$. b) What is the period of the tide, from one high tide to the next? c) A bulk carrier needs at least 14.5 m of water to dock safely. For how many hours per cycle can the bulk carrier dock safely?
		10.3 The average daily maximum temperature in Vancouver follows a sinusoidal pattern with a highest value of 23.6°C on July 26, and a lowest value of 4.2°C on January 26. a) Describe this variation with a sine or cosine equation. b) What is the expected maximum temperature for May 26? c) How many days will have an expected maximum of 21.0°C or higher? d) Explain why different equations give the same answers for b) and c).

Strand: Shape and Space (Transformations)

Students will:

- perform, analyze and create transformations.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[E]	Estimation and Mental Mathematics	**[T]**	Technology
		[V]	Visualization

General Outcomes	Specific Outcomes		Illustrative Examples
(continued)	P9–6. (SS40)	Describe how reflections of functions in both axes and in the line $y = x$ affect graphs and their related equations: • $y = f(-x)$ • $y = -f(x)$ • $y = f^{-1}(x)$. [C, T, V]	6.1 Graph any function $f(x)$. Sketch the graph of: a) $-f(x)$ b) $f(-x)$ c) $f^{-1}(x)$ d) $f^{-1}[f(x)]$. 6.2 If $g(x)$ is the reflection of $f(x)$ in the y-axis, write the equation of $g(x)$ in terms of $f(x)$.
	P9–7. (SS41)	Using the graph and/or the equation of $f(x)$, describe and sketch $\frac{1}{f(x)}$. [C, T, V]	7.1 Given $f(x) = 2x + 1$, sketch the graph of $f(x)$ and of $\frac{1}{f(x)}$. What happens to the x-intercepts of $f(x)$? 7.2 Sketch the graph of $f(x) = \sin x$, and sketch $\frac{1}{\sin x}$. 7.3 Sketch $\frac{1}{f(x)}$, if $f(x)$ is shown by the accompanying sketch. 

Strand: Shape and Space (Transformations)

Students will:

- perform, analyze and create transformations.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[E]	Estimation and Mental Mathematics	**[T]**	Technology
		[V]	Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
Perform, analyze and create transformations of functions and relations that are described by equations or graphs.	P9–4. (SS38) Describe how various translations of functions affect graphs and their related equations: • $y = f(x - h)$ • $y - k = f(x)$. [C, T, V]	4.1 Describe how the graph of $y = x^2$ compares to the graph of $y = x^2 - 2$. 4.2 Graph any function $f(x)$. On the same set of coordinate axes, sketch the graph of: a) $f(x) - 2$ b) $f(x - 2)$ c) $f(x - 2) + 1$.
	P9–5. (SS39) Describe how various stretches of functions (compressions and expansions) affect graphs and their related equations: • $y = af(x)$ • $y = f(kx)$. [C, T, V]	5.1 Describe how the graph of $y = x^2$ compares to the graph of: a) $y = 2x^2$ b) $y = \frac{2}{3}x^2$. 5.2 Graph any function $f(x)$. On the same set of coordinate axes, sketch the graph of: a) $2f(x)$ b) $-2f(x)$ c) $\frac{2}{3}f(x)$. Discuss the changes. 5.3 Given the graph of $f(x) = \sin x$, sketch the graph of: a) $f(2x)$ b) $\frac{2}{3}f(x)$. 5.4 Given the graph of $f(x) = x^3$ and its image under the transformation $g(x) = 3f(x)$, find the equation describing $g(x)$.
(continued)		

Strand: Statistics and Probability (Chance and Uncertainty)

Students will:

- use experimental or theoretical probability to represent and solve problems involving uncertainty.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[E]	Estimation and Mental Mathematics	**[T]**	Technology
		[V]	Visualization

<table>
<tr><th>General Outcomes</th><th colspan="2">Specific Outcomes</th><th colspan="2">Illustrative Examples</th></tr>
<tr><td rowspan="2">Use normal and binomial probability distributions to solve problems involving uncertainty.

<i>(continued)</i></td><td>C6–1.
(SP11)</td><td>Find the population standard deviation of a data set or a probability distribution, using technology.
[CN, E, T, V]</td><td>1.1

1.2</td><td>Measure the height of each student in a class, and calculate the mean and standard deviation.

A company uses an automated packaging device to produce 50-g bags of Karmel Korn. The machine needs frequent checking to see if it is actually putting 50 g in each bag. The following are the masses, in grams, of thirty bags of Karmel Korn.
<table>
<tr><td>54</td><td>50</td><td>47</td><td>50</td><td>51</td><td>50</td></tr>
<tr><td>53</td><td>50</td><td>47</td><td>51</td><td>50</td><td>51</td></tr>
<tr><td>52</td><td>49</td><td>46</td><td>52</td><td>50</td><td>49</td></tr>
<tr><td>52</td><td>48</td><td>48</td><td>53</td><td>49</td><td>49</td></tr>
<tr><td>51</td><td>48</td><td>49</td><td>52</td><td>49</td><td>50</td></tr>
</table>
a) Calculate the mean and standard deviation of this data.
b) What problems will be encountered, if the standard deviation gets too high?

Dottori et al., <i>Foundations of Mathematics 11</i>, p. 392. Adapted with permission.</td></tr>
<tr><td>C6–2.
(SP12)</td><td>Use <i>z</i>-scores and <i>z</i>-score tables to solve problems.
[PS, R, T, V]

<i>(continued)</i></td><td>2.1</td><td>The volume of the contents of a soft drink can is normally distributed about a mean of 350 mL, with a standard deviation of 1.5 mL.
a) Calculate the <i>z</i>-score for a can with a volume of 355 mL.
b) What percentage of production will consist of cans having content volumes between 350 mL and 355 mL?
c) What percentage of production will consist of cans having content volumes less than 355 mL?
d) If cans containing less than 346 mL must be rejected, how many cans will be expected to be rejected in a run of 50 000?</td></tr>
</table>

Strand: Shape and Space (Transformations)

Students will:

- perform, analyze and create transformations.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[E]	Estimation and Mental Mathematics	**[T]**	Technology
		[V]	Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
(continued)	P9–8. (SS42) Using the graph and/or the equation of $f(x)$, describe and sketch $\lvert f(x)\rvert$. [C, T, V]	8.1 Given the graph of $f(x) = 2x + 1$, sketch $\lvert f(x)\rvert$. 8.2 Sketch $y = 3\lvert\sin x\rvert$. What is the period of this function? 8.3 Sketch $f(x) = \dfrac{1}{\lvert x^2 - 1\rvert}$. 8.4 An AC generator has a voltage given by $V = 170\cos(120\pi t)$, where V is the voltage and t the time in seconds. A simple DC rectifier has voltage output given by $V = 170\lvert\cos(120\pi t)\rvert$. Sketch the output graphs for both devices, and describe the similarities and differences.
	P9–9. (SS43) Describe and perform single transformations and combinations of transformations on functions and relations. [C, T, V]	9.1 Given $f(x) = x^2$, sketch the graph of $f(x)$, and sketch the graph of $-2f(x - 1) + 3$. 9.2 Determine the equation of the ellipse $x^2 + 4y^2 - 25 = 0$, after each of the following transformations: a) translated two units to the right b) translated three units down c) expanded by a factor of two along the horizontal axis d) expanded by a factor of one quarter along the vertical axis. 9.3 Given the circle $x^2 + y^2 = 1$ and its image under a translation described by the ordered pair $(2, -3)$: a) write the equation of the image b) if a point $P(a, b)$ is on the graph of the circle $x^2 + y^2 = 1$ and $P'(a', b')$ is the transformed image of P, what are the coordinates of P' in terms of a and b?

Strand: Statistics and Probability (Chance and Uncertainty)

Students will:

- use experimental or theoretical probability to represent and solve problems involving uncertainty.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[E]	Estimation and Mental Mathematics	**[T]**	Technology
		[V]	Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
(continued)	C6–3. (SP13) Use the normal distribution and the normal approximation to the binomial distribution to solve problems involving confidence intervals for large samples. [CN, E, PS]	3.1 The heights of males employed by a manufacturer follow a normal distribution with a mean of 169 cm and a standard deviation of 8 cm. a) Establish a symmetric 95% confidence interval for the average height in a random sample of 36 male employees. b) What happens to the width of the symmetric 95% confidence interval, if the sample size is increased from 36 to 225? 3.2 Pollsters estimate that the number of decided voters in favour of a particular bylaw is 64%, and the number opposed is 36%. a) If the sample size is 250, find the expected mean and standard deviation of *yes* voters. b) Estimate, for this sample, the expected percentage of *yes* voters, with a symmetric 95% confidence interval used to establish the margin of error. c) If the margin of error for the percentage of *yes* voters must be less than ±1.0%, what would be the minimum sample size required? 3.3 The probability that a car salesperson will complete a sale is 0.10. If the salesperson has 200 customers in the next month, establish a symmetric 95% confidence interval for the number of completed sales for the month.

Strand: Statistics and Probability (Chance and Uncertainty)

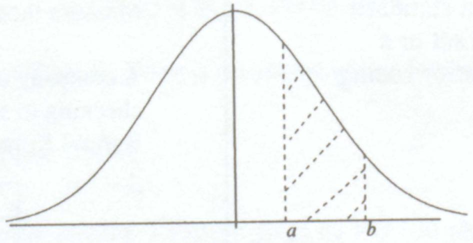
Students will:

- use experimental or theoretical probability to represent and solve problems involving uncertainty.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[E]	Estimation and Mental Mathematics	**[T]**	Technology
		[V]	Visualization

<table>
<tr><th>General Outcomes</th><th>Specific Outcomes</th><th>Illustrative Examples</th></tr>
<tr><td><i>(continued)</i></td><td><i>(continued)</i></td><td>

2.2



a) What is the area under this curve?
b) If $P(a < z < b) = 0.4$, what is the area under the curve for the interval $a < z < b$?
c) If $P(z < b) = 0.9$, calculate $P(z > b)$, and calculate the value of b.

2.3 For entry into the Canadian Armed Forces, the standards for height used to be set at 158 cm to 194 cm for males, and 152 cm to 184 cm for females. Use the concept of z-score to test if these two height standards are equivalent. Assume means of 176 cm and 163 cm and standard deviations of 8 cm and 7 cm respectively.

2.4 A sample of 122 people gives a mean body temperature of 36.8°C, with a standard deviation of 0.35°C. Assuming a normal distribution, find:
a) the expected number of people with temperatures above 37.0°C
b) the expected number of people with temperatures below 36.0°C.

Also, estimate the range of temperatures contained within the sample.

2.5 In the general population, the IQ scores of individuals is normally distributed with a mean of 110 and a standard deviation of 10. If a large group of people is tested:
a) What proportion of this group is expected to have IQs between 100 and 120?
b) What is the probability that an individual in the group has an IQ greater than 120?

</td></tr>
</table>

Strand: Statistics and Probability (Chance and Uncertainty)

Students will:

- use experimental or theoretical probability to represent and solve problems involving uncertainty.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[E]	Estimation and Mental Mathematics	**[T]**	Technology
		[V]	Visualization

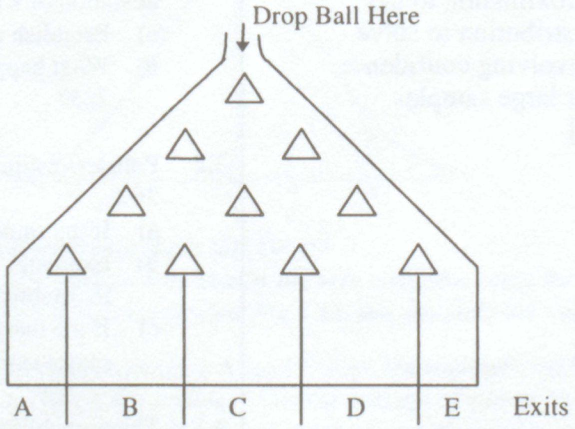
General Outcomes	Specific Outcomes	Illustrative Examples
Solve problems based on the counting of sets, using techniques such as the fundamental counting principle, permutations and combinations.	P7–12. (SP16) Determine the number of permutations of n different objects taken r at a time, and use this to solve problems. [PS, R, V]	12.1 List all possible permutations of the letters in the word **bold**. 12.2 Calculate the number of ways that an executive consisting of four people (president, vice-president, treasurer and secretary) can be selected from a group of 20 people. 12.3 Explain the meaning of ${}_8P_3$. Why does ${}_3P_8$ not make sense? 12.4 Develop and solve a problem where ${}_8P_3$ would be applicable. 12.5 Solve ${}_nP_2 = 30$. 12.6 On a 12-question multiple-choice test, three answers are A, three are B, three are C and three are D. How many different answer keys are possible?
(continued)	P7–13. (SP17) Determine the number of combinations of n different objects taken r at a time, and use this to solve problems. [PS, R, V]	13.1 From a group of five student representatives, three will be chosen to work on the dance committee. a) List all possible committees. b) Calculate ${}_5C_3$, and compare to the answer in part a). c) If the committee had to have a chairperson, would it still be a combination? Why or why not? d) How many committees of three, with a chairperson, can be chosen from a group of 10 student representatives? 13.2 Show that ${}_nC_k = {}_nC_{(n-k)}$, using two different methods. Verify the truth of this assertion for the special case with $n = 10$ and $k = 3$. 13.3 How many diagonals are there in a regular polygon with 20 sides? What is the general formula for the number of diagonals in an n-sided polygon?

Strand: Statistics and Probability (Chance and Uncertainty)

Students will:

- use experimental or theoretical probability to represent and solve problems involving uncertainty.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
Solve problems based on the counting of sets, using techniques such as the fundamental counting principle, permutations and combinations.	C6–4. (SP14) Solve pathway problems, interpreting and applying any constraints. [PS, R]	4.1 Given the following "pinball" situation, what is the probability of the ball reaching each of the exits?  What assumptions are made in the solution?
	C6–5. (SP15) Use the fundamental counting principle to determine the number of different ways to perform multistep operations. [PS, R]	5.1 Joe has three different shirts, two different pairs of pants and five different pairs of shoes. List all possible outfits in such a way as to ensure that all have been counted and none have been counted twice. How many possible outfits are there? Use the fundamental counting principle to determine the number of outfits there should be. Do your answers match? 5.2 An airline pilot reported that in seven days she spent one day in Winnipeg, one day in Regina, two days in Edmonton and three days in Yellowknife. How many different itineraries are possible? What difference would it make if the first day and the last day had to be spent in Yellowknife?

Strand: Statistics and Probability (Chance and Uncertainty)

Students will:

- use experimental or theoretical probability to represent and solve problems involving uncertainty.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[E]	Estimation and Mental Mathematics	**[T]**	Technology
		[V]	Visualization

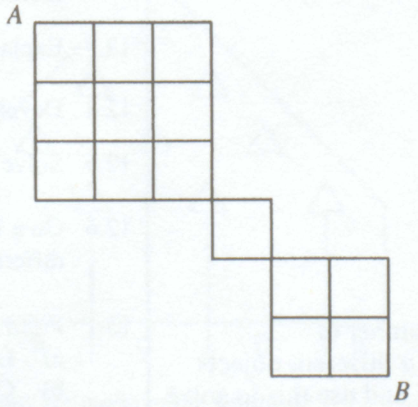
General Outcomes	Specific Outcomes	Illustrative Examples
(continued)	P7–15. (SP19) Solve problems, using the binomial theorem where N belongs to the set of natural numbers. [CN, E, V]	15.1 Expand $(x + y)^7$, using the binomial theorem. 15.2 Find the 11th term of the expansion of $(x - 2)^{13}$. 15.3 Investigate the sample space for flipping 1 coin, 2 coins, 3 coins, 4 coins . . . , and make an organized list. Relate this organized list to Pascal's triangle and the binomial theorem. 15.4 Given a set of four elements, list the different proper and improper subsets, and organize them. How is this related to Pascal's triangle? How many subsets are there in total?

Strand: Statistics and Probability (Chance and Uncertainty)

Students will:

- use experimental or theoretical probability to represent and solve problems involving uncertainty.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
(continued)	P7–14. (SP18) Determine the number of pathways in a given compound pathway problem. [CN, PS, V]	14.1 Student *A* wants to visit Student *B*. Roads are shown as lines on the grid. Only south and east travel directions can be used.  a) How many different paths can *A* take to get to *B*, if *A* has to travel along the lines that represent the roads? b) Change the middle square to a 2×2 grid, and repeat the question.

Strand: Statistics and Probability (Chance and Uncertainty)

Students will:

- use experimental or theoretical probability to represent and solve problems involving uncertainty.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[E]	Estimation and Mental Mathematics	**[T]**	Technology
		[V]	Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
Model the probability of a compound event, and solve problems based on the combining of simpler probabilities.	P7–16. (SP23) Determine the conditional probability of two events (Bayes' law). [E, PS, R]	16.1 In a particular country, the probability of a child being a girl is 0.510. A family of five children is known to have at least two girls. What is the probability of this family having exactly four girls? 16.2 It is known that 10% of a population has a certain disease. For a patient without the disease, a blood test for the disease gives a correct diagnosis 95% of the time. For a patient with the disease, the test gives a correct diagnosis 99% of the time. What is the probability that a person whose blood test shows the disease actually has the disease?
	P7–17. (SP24) Solve probability problems involving permutations, combinations and conditional probability. [E, PS, R]	17.1 Five books, each of a different colour, and including one red and one green book, are placed on a shelf. What is the probability of the red book being at one end and the green book at the other? 17.2 What is the probability of holding all four aces in a five-card hand dealt from a standard 52-card deck? 17.3 A shootout consists of teams A and B taking alternate shots on goal. The first team to score wins. Team A has a probability of 0.3 of scoring with any one shot. Team B has a probability of 0.4 of scoring with any one shot. a) If Team A shoots first, what is the probability of Team B winning on its first shot? b) If Team A shoots first, what is the probability of Team A winning on its third shot? c) What is the probability of Team A eventually winning? d) If Team B shot first, what is the probability of Team B eventually winning?
	P7–18. (SP25) Solve probability problems, using the binomial distribution as applied to small samples. [PS, R, T]	18.1 A written test for a driver's licence consists of 10 multiple-choice questions. To pass the test, a driver must answer 9 or 10 questions correctly. What is the probability of passing by guessing, if there are four possible answers to each question? 18.2 A family has nine children. Assuming that there is an equal likelihood for male and female births, what is the probability that there are seven boys and two girls? 18.3 An 8 km/h crash test was given to a sample of 20 cars. Four cars failed the test because of damaged bumpers. Find a 95% confidence interval for the proportion of cars that would fail this crash test.

Strand: Statistics and Probability (Chance and Uncertainty)

Students will:

- use experimental or theoretical probability to represent and solve problems involving uncertainty.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[E]	Estimation and Mental Mathematics	**[T]**	Technology
		[V]	Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
Model the probability of a compound event, and solve problems based on the combining of simpler probabilities.	C6–6. (SP20) Construct a sample space for two or three events. [PS, R, V]	6.1 List the sample space for rolling a 6-sided die and flipping a coin. 6.2 Draw or list the sample space for the following situation. A bus is scheduled to arrive at a train station at any time between 07:05 and 07:15 inclusive. A train is scheduled to arrive between 07:11 and 07:17 inclusive. The arrival of a bus at 07:06 and a train at 07:14 can be represented by the point (6, 14). Times are expressed in whole minutes. a) How many points are there in this sample space? b) How many points have the bus and the train arriving at the same time? c) How many points have the bus arriving after the train? d) What is the probability of the bus arriving after the train?
	C6–7. (SP21) Classify events as independent or dependent. [C]	7.1 Classify the following events as independent or dependent: a) tossing a head in a coin toss and rolling a 6 on a die b) drawing an ace for the first card and another ace for the second, if the experiment is carried out without replacement c) drawing a king for the first card and a queen for the second, if the experiment is carried out with replacement. 7.2 Sixty per cent of young drivers take driver training, and 25% of young drivers have an accident in their first year of driving. Statistics show that 10% of those who do take driver training have an accident in their first year. Are taking driver training and having an accident in the first year independent events?
	C6–8. (SP22) Solve problems, using the probabilities of mutually exclusive and complementary events. [CN, PS, R]	8.1 If the probability of winning a game is $\frac{1}{31}$, what is the probability of losing the game? 8.2 A shootout consists of teams A and B taking alternate shots on goal. The first team to score wins. Team A has a probability of 0.3 of scoring with any one shot. Team B has a probability of 0.4 of scoring with any one shot. a) If Team A shoots first, what is the probability of Team B winning on its first shot? b) If Team A shoots first, what is the probability of Team A winning on its third shot?